VOLUME LVIII NUMBER 3

OCT 26 1923

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

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The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, El Salvador, Argentina, Bolivia, Brazil, Colombia, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Uruguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, and Spain. ¶Postage is charged extra as follows: for Canada, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies 5 cents (total 80 cents). ¶Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12*s.* 6*d.* each; single copies, including postage, 4*s.* each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 13.65 each; single copies, including postage, Yen 1.68 each.

For China: The Mission Book Company, 13 North Szechuen Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Ryerson Laboratory, University of Chicago, Chicago, Illinois.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the Act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE
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VOLUME LVIII OCTOBER 1923 NUMBER 3

ON THE DOUBLE STAR 9 ARGUS

By OTTO STRUVE

ABSTRACT

Observations of the visual double star 9 Argus, made with the Bruce spectrograph of the Yerkes Observatory, were used for finding the *sign of the inclination* and the actual *dimensions of the orbit*. The period of revolution of this system is 23.34 years. A comparison of the spectrographic results with the visual orbit (by Aitken) gives $\pi m_1/m = 0''.14$. The most probable value of the *absolute parallax*, obtained by the trigonometric and other methods, is $\pi = 0''.078$. The *mass-ratio* is approximately $m/m_1 = 0.6$. The *total mass of the system* is found to be near 1.0–1.1 ⊙. This is in good agreement with the value 1.2 ⊙ obtained by Miller and Pitman using the law of gravitation.

The star 9 Argus, known as β 101 ($\alpha = 7^h47^m.1$, $\delta = -13°38'$ (1900); mag., 5.6 and 6.7; spectral type, F8)[1] was placed by Professor Edwin B. Frost on the observing program for radial velocity at the Yerkes Observatory in 1905. The intention was to accumulate data for determining the sign of the inclination and the absolute parallax (where possible) for a number of visual binaries having well-established orbits. Since that time eleven spectrograms of this star, of good quality, have been obtained. All the plates have been measured and reduced by the usual Hartmann formula. Most of the plates were measured independently by different observers and no plate was measured less than twice. The spectrum of the fainter component does not appear on our plates and the measures

[1] In the *Henry Draper Catalogue*, for the brighter component, the spectral type is given as Go and the combined visual or "photometric" magnitude as 5.34.

refer only to the brighter component. Table I contains the results of the measurements. The means of the various values for each plate were adopted as final and are given in the last column.

The visual orbit of the double star 9 Argus is known with great accuracy. A number of orbits have been published for this star. The best is undoubtedly the one by R. G. Aitken,[1] though his elements differ only slightly from the elements computed by Burnham[2]

TABLE I

Plate	Date	G.M.T.	Taken By	Velocity	Meas. By	Mean
				km/sec		km/sec
1B 493....	1905, Jan. 27	19h06m	F, B, S	−17.6, −15.8, −13.3	F, P, L	−15.6
11B 89....	1906, Nov. 23	21 42	B, S	−16.5, −14.6	L, σ	−15.5
1B 933....	1906, Dec. 17	20 14	B, S	−17.5, −17.9, −20.6	F, P, L	−18.7
1B 939....	1907, Jan. 4	19 38	F, B, S	−22.9, −22.6, −18.2, −23.4	F, P, L, σ	−21.8
1B 1322....	1908, Jan. 13	19 30	L, B, S	−20.0, −22.2, −20.2	P, M, L	−23.8
1B 4789....	1917, Jan. 29	15 40	B	−19.5, −19.5	M, σ	−19.5
1B 6376....	1922, Jan. 27	18 37	σ, S	−13.1, −16.0	σ, σ	−14.6
1B 6384....	1922, Jan. 30	19 09	σ, S	−14.4, −18.3	σ, σ,	−16.4
1B 6465....	1922, April 7	14 48	σ, S	−14.7, −18.9	σ, σ	−16.8
1B 6653....	1922, Nov. 6	22 09	σ, S	−12.8, − 8.0	σ, σ,	−10.4
B 966....	1922, Nov. 20	20 31	σ, S	−17.3, −16.7	σ, σ,	−17.0

F=E. B. Frost, B=S. B. Barrett, P=J. A. Parkhurst, L=O. J. Lee, M=C. A. Maney, S=F. R. Sullivan, σ=O. Struve.

TABLE II

Ephemeris by Aitken			Observations			
	Pos. Angle	Distance		Pos. Angle	Distance	
1917.25...	280°.3	0″.40				
1918.25...	284.3	0.53	1918.18..	289°.1	0″.48	Van Biesbroeck
1919.25...	287.0	0.59	1919.11..	292.6	0.54	Van Biesbroeck
1920.25...	289.3	0.62				
1921.25...	291.5	0.62	1921.14..	299.9	0.52	Van Biesbroeck
1922.25...	293.8	0.62	1922.19..	293.4	0.56	Paraskévopoulos
			1923.27..	304.5	0.62	Van Biesbroeck

and later by Lohse.[3] Unpublished observations made by Professor G. Van Biesbroeck with the 40-inch telescope of the Yerkes Observatory and by Dr. J. Paraskévopoulos with the 16-inch telescope at Athens, as compared with the computed ephemeris, prove that the orbit, though derived in 1912, still represents the observations with sufficient accuracy.

[1] *Publications of the Lick Observatory*, **12**, 51.

[2] Burnham's *General Catalogue*, No. 4310, p. 504.

[3] *Publicationen des Astrophys. Observatoriums*, Potsdam, **20**, 93.

It seemed therefore unnecessary to undertake a new determination of the visual orbit, and Aitken's elements were adopted without change.

ELEMENTS

$P=23.34$ years	$a=\ 0''.69$
	$w=\ 74°.65$
$T=1892.60$	$i=\pm 79°.8$
$e=\ 0.75$	$\Omega=\ 99°.7$
	Angles increasing

The inclination i is positive since the companion is being carried away from the observer at the nodal point.

In order to combine the spectrographic observations with the visual orbit, for each spectrographic velocity an equation of the following form was computed:

$$\frac{d\zeta}{dt}=V+\frac{dz}{dt}$$

$$\frac{dz}{dt}=\frac{\mu a_1 \sin i}{\sqrt{1-e^2}}[e\cos\omega+\cos(v+\omega)]$$

The notation is the same as in Aitken's *The Binary Stars*, pages 72 and 135–136.

It should be remembered that the spectrographic velocities are referred to the center of gravity of the system, while the visual orbit refers to the brighter component as the center of the coordinates.

Therefore we shall call a the semimajor axis of the visual orbit, while a_1 is the semimajor axis of the spectrographic orbit.

These equations were solved by the method of least squares and gave the following normal equations:

$$1.\ 3779x+\ 2.3185V+36.5127=0$$
$$2.\ 3185x+11.0000V+188.1000=0$$

where

$$x=\frac{\mu a_1 \sin i}{\sqrt{1-e^2}}$$

and V is the velocity of the center of mass of the system.

Solving these equations, we obtain:

$$V=-17.8 \text{ km/sec} \qquad x=3.5241.$$

No probable errors were computed for these quantities, for they would be entirely illusory, since their accuracy is based mainly on the accuracy of the visual orbit.

Remembering that

$$\mu = \frac{2\pi}{P_{\text{seconds}}}$$

we find $a_1 = 300{,}000{,}000$ km, while from the visual orbit $a = 0''.69$.

If the masses of the two components were equal, we would have

$$a = 2a_1$$

If the mean distance of the earth from the sun is 150,000,000 km, then we would have:

$$\pi = \frac{0''.69 \times 150}{600} = 0''.17.$$

In the case of 9 Argus the mass-ratio is not known with sufficient accuracy and may differ considerably from unity. Therefore we find only:

$$\pi\left(\frac{m_1}{m}\right) = 0''.17.$$

We could solve our problem in a somewhat different way. If we draw a smooth curve through the spectrographic observations, retaining only the period of the visual orbit, we would find a range of 16 km/sec between maximum and minimum velocities.

Using the equation

$$a_1 \sin i = \text{const.} \frac{\kappa\sqrt{1-e^2}}{\mu}$$

we find

$$a = 2a_1 = 900{,}000{,}000 \text{ km}.$$

This gives

$$\pi\left(\frac{m_1}{m}\right) = 0''.11.$$

Taking the mean between the two values $0''.17$ and $0''.11$ we have finally

$$\pi\left(\frac{m_1}{m}\right) = 0''.14.$$

Below is given a list of other determinations of the parallax of 9 Argus:

(1) Flint	$+0''.035 \pm 0''.026$	Meridian circle
(2) Sproul	$+0.121 \pm 0.009$	Photography
(3) McCormick	$+0.036 \pm 0.008$	Photography
(4) Adams and Joy	$+0.079$	Spectroscopic method
(5) Jackson and Furner	$+0.067$	Hypothetical value

Using only the best determination (2–5) and adding $0''.005$ to the values obtained by the trigonometric method, we adopt for the absolute parallax

$$\pi = 0''.078.$$

Combining this value with the one found from the spectrographic observations, we obtain for the mass-ratio approximately:

$$\frac{m}{m_1} = 0.6.$$

Considering the different values for the parallax of this star, as determined by the trigonometric method, we must admit that the discrepancy between the results obtained by J. A. Miller at the Sproul and by S. A. Mitchell at the McCormick observatories is rather unusual. The epochs of the parallax plates were the same at the two observatories, i.e., 1915–1916–1917. It seems not impossible that the photographic images of the parallax star have been affected by the light of the companion, which was in periastron in 1915.9 and which was moving very rapidly in position angle between the years 1915 and 1917 (99°–280°). Dr. O. J. Lee suggests that the large difference between the two values of π may be due to the circumstance that the light of the companion was not entirely eliminated by cutting down the brightness of the parallax star by means of a rotating sector, and that probably the effect of the sector was different at the two observatories. Therefore the light of the companion might have affected the image of the parallax star more in the one case than in the other.

In spite of this discordance, there is no doubt as to the reality of the parallax of 9 Argus, and it seems highly probable that the adopted value $0''.078$ is very close to the truth.

S. A. Mitchell[1] finds from a comparison of proper motions as given by Boss and by his own determinations that the masses of the two components are in the ratio 0.73 to 0.27, or approximately

$$\frac{m}{m_1}=0.4.$$

This value agrees fairly well with the mass-ratio found from the spectrographic observations,

$$\frac{m}{m_1}=0.6.$$

In addition it may be noted that the mean density of this system was computed by E. Öpick,[2] and later, on the basis of better observational material by E. Bernewitz.[3] The two values are

$$\delta=2.3 \quad \text{and} \quad \delta=1.01,$$

the unit being the density of the sun.

Though differing very considerably from each other they still indicate that the density of this system is rather high.

If we adopt with Burnham the apparent magnitudes of the two components as, respectively, 5.6 and 6.7, then the absolute magnitude of the brighter component would be 5.1 (for $\pi=0''.078$). H. N. Russell's tables[4] give for the diameter of such a star

$$D=0.9,$$

expressed in diameters of the sun. This would correspond to about 0.7 of the volume of the sun.

Applying Bernewitz's value for the mean density of the star, i.e., 1.01, we find that the mass of the brighter component is 0.7 that of the sun.

If our value for the mass-ratio (0.6) is correct, then the mass of the fainter component will be 0.4 that of the sun and

$$m+m_1=1.1\ \odot.$$

[1] *Publications of the Leander McCormick Observatory*, **3**, 239.

[2] *Astrophysical Journal*, **44**, 298, 1916.

[3] *Astronomische Nachrichten*, **213**, 1, 1921.

[4] *Publications of the Astronomical Society of the Pacific*, **32**, 307, 1920.

This result agrees remarkably well with the value of the total mass of the system, derived from Kepler's law by Miller and Pitman[1] and independently by B. Meyermann.[2] Their values are identical, being based on the same parallax ($\pi = 0''.079$) and give

$$m + m_1 = 1.2.$$

Knowing the mass-ratio, we could compute the density of the brighter component from Bernewitz' formula (1):

$$\log \delta_1 = \log\left(\frac{a^3}{u^2}\right) + 0.6(m_1 - i_1) - \log\left(1 + \frac{m}{m_1}\right) + 0.089.$$

The computation gives

$$\delta_1 = 0.8.$$

Hence approximately

$$m_1 = 0.6\ \odot, \qquad m = 0.4\ \odot$$
$$m + m_1 = 1.0\ \odot.$$

It must be remembered that the value for $m + m_1$, derived from the law of gravitation, depends on the third power of π. Therefore, a small error in the adopted parallax produces a great uncertainty in the mass. The good agreement between our hypothetical values of $m + m_1$ and the one derived by Miller and Pitman is probably the best proof that the adopted parallax $0''.078$ or $0''.079$ is not far from its true value. On the other hand, it must be admitted that the hypothetical value for the diameter, being true in average, may be subject to considerable uncertainty if applied to a particular star.

At the present time, the radial velocity of 9 Argus is slowly decreasing, the brighter component being carried away by its orbital motion with decreasing speed. Since the velocity of the system as a whole is -17.8 km /sec, the resulting velocity is always negative. A very rapid change from minimum to maximum velocity will occur in the years 1938–1941, and it will be of great value to secure a number of good spectrograms at that time.

[1] *Astronomical Journal*, **34**, 127, 1922.

[2] *Astronomische Nachrichten*, **216**, 302, 1922.

Since the completion of the preceding work in April, 1923, special experiments have been made by the writer with the Bruce spectrograph to prove that the observed spectrum of 9 Argus was affected by the light of the brighter component only.

It is now proved that, in a composite spectrum, the lines of the fainter component are entirely invisible if the difference in brightness of the two components equals or exceeds 1.0 magnitude. For a difference in brightness of 0.5 magnitude, the intensity of the lines of the fainter component is only about 15 per cent of the intensity of the lines of the brighter star. If the difference in magnitude of the two components of a close double star is less than 1.0 magnitude, and the resolving power of the spectrograph not sufficient for separating the two lines, it would be necessary to apply corrections, to the measured velocities, for the effect of blending with the lines of the fainter component. In the case of 9 Argus the difference in brightness exceeds 1.0 magnitude and the correction is therefore negligible.

YERKES OBSERVATORY
June 30, 1923

A DETERMINATION OF e/m FROM MEASUREMENTS OF THE ZEEMAN EFFECT[1]

By HAROLD D. BABCOCK

ABSTRACT

The methods used in the determination of the ratio e/m.—Forty-nine separate values were derived from spectroscopic observations of the Zeeman effect, with absolute measurements of the magnetic field-intensity. About 40 lines in the blue region of the chromium spectrum were most frequently used. These were supplemented by 76 other lines of chromium, titanium, zinc, and barium. At *field-strengths averaging about 30,000 gausses* the separations of these lines were measured for the derivation of the normal separation, a. A wide variety of types of separation was used, but most dependence was placed on lines known to have small denominators in their fractional relation to a. Field measurements were made with ballistic galvanometer, test coil, and mutual inductance, in the primary circuit of which a known current could be reversed. *The uncertainty* affecting each factor in the working equation is discussed and the combined effect of all upon the result is derived. The chief difficulty in this method is the determination of a, the only other source of appreciable error being the magnetic area of the test coil.

Weighted mean e/m.—The final result from all the observations is $e/m = 1.761 \times 10^7$, with a probable error of one part in 1800 and an estimated actual uncertainty of 2 or 3 parts in 1800.

INTRODUCTION

Among the numerous determinations which have been made of the ratio e/m there is found a considerable discrepancy. Many of the older observations are so obviously inaccurate as to be of little value, while, even among those results which are apparently the best, the differences are objectionably large. Webster and Page,[2] using Paschen's data for hydrogen and helium series, find $e/m = 1.7686 \times 10^7$, which they say "is probably more accurate than any other determination." They do not give the probable error of this result. Birge,[3] on the other hand, using the most recent experimental results for hydrogen and helium, finds $e/m = 1.758 \times 10^7$, with a probable error of fully 0.5 per cent. He points out the desirability of new measurements based on deflection methods or on the Zeeman effect.

[1] *Contributions from the Mount Wilson Observatory*, No. 263.

[2] "A General Survey of the Present Status of the Atomic Structure Problems," *Bulletin of the National Research Council*, **2**, 368 (No. 14), 1921.

[3] *Nature*, **111**, 287, 1923.

The observations to be described were made during the progress of an investigation of the Zeeman effect for iron, chromium, and vanadium which is not yet published. The large amount of observational material thus collected has now been summarized, and this has made feasible the derivation of the results given here.

METHODS AND INSTRUMENTS

From the fundamental equation for the Zeeman effect we have:

$$\frac{e}{m} = \frac{4\pi V}{H}\frac{\Delta\lambda}{\lambda^2}$$

where H = intensity of magnetic field

$\Delta\lambda$ = separation of either π-component of a normal triplet of wavelength λ, from the undisturbed position of the line.

V = velocity of light in the medium in which $\Delta\lambda$ and λ are measured.

The quantity $\Delta\lambda/\lambda^2$ will be replaced throughout this paper by the symbol a.

The procedure which has been followed is to measure a for a selected group of lines of different elements, making absolute determinations of H, and to assume for V the value 2.9978×10^{10} cm/sec. In their important paper on the ratio of the electromagnetic to the electrostatic unit of the electricity, Rosa and Dorsey[1] conclude that the most probable value of the velocity of light *in vacuo* is 2.9986×10^{10} cm/sec, with an uncertainty of 1 part in 10,000. The index of refraction of air under normal conditions for λ 4500 is given by Meggers and Peters[2] as 1.00028. Since this number represents the ratio of the velocity of light *in vacuo* to its velocity in air with sufficient accuracy over the range of working conditions encountered in the present work, it is used for deriving the single value of V which enters into the equation for evaluating e/m.

The values of a were determined from photographs made with the vertical Littrow spectrograph in the Pasadena Laboratory. Second- and third-order spectra were used, sometimes at the 13-foot (4 m) focus, but nearly always at the 30-foot (9.2 cm) focus. A majority of the observations was made with a grating ruled by

[1] *Bulletin of the Bureau of Standards*, **3**, 541, 1907.

[2] *Ibid.*, **14**, 731, 1919.

Anderson at the Johns Hopkins University. This grating has 42,386 lines and was found to give its theoretical resolving power in the second and third orders. Two other gratings, one ruled by Rowland and one by Michelson, were used occasionally.

The slit width was calculated by dividing the mean wave-length for a given photograph by the angular width of the ruled surface as seen from the slit. This gives four times the normal slit width as defined by Schuster,[1] and proves a satisfactory compromise between the opposing claims of intensity and resolving power.

The light was produced by the discharge of a condenser supplied with power from an alternating current transformer. The energy flowing into the transformer on the low-tension side, as measured with an indicating wattmeter, was never less than one kilowatt nor more than one and one quarter kilowatts. Inductance was always added to the discharge circuit to improve the sharpness of the spectral lines and to reduce the intensity of the air spectrum. The spark terminals consisted of lumps or rods of metal held in a non-magnetic clamp. Chromium, titanium, brass, and cadmium were used. In some cases the two terminals were different but more often they were of the same metal. Occasionally a solution of barium bromide was dropped upon the metal terminals, thus giving the strong barium line λ 4554 in addition to the spectrum of the terminals themselves.

Table I gives the lines which were used in this work, the elements to which they belong, and the fractions which indicate the part of a associated with each component. For the spectral lines under consideration the components occur in pairs symmetrical about the normal position of the line, except that in the case of lines having an odd number of components the middle p-component is undisplaced. This is indicated by 0 in the numerator of the fraction. Parentheses inclose each group of p-components, and in the case of lines having normal separation the denominator 2 is arbitrarily assigned. The fraction would be written $(0)2/2$ for lines of this kind. The sign $\pm$ is always understood to precede the fractions as written.

[1] *Astrophysical Journal*, **21**, 207, 1905

TABLE I

SPECTRAL LINES USED FOR DETERMINATION OF a

λ	Element	Fractional Separation	No. of Plates	λ	Element	Fractional Separation	No. of Plates
3841.32	Cr	(?), 6/7	1	4280.43	Cr	(0), 20/21	1
3852.23	Cr	(0), 3/2	1	4289.73	Cr	(2, 1, 0), 8, 7, 6, 5, 4/3	6
3857.66	Cr	(26), 53, 27/21	1	4295.79	Cr	(0), 5/3	4
3865.57	Cr	(0), 11/10	1	4297.78	Cr	(0), 12, 0?/13	4
3883.34	Cr	(0), 3/2	3	4300.53	Cr	(0), 2/2	1
3885.25	Cr	(0), 3/2	3	4301.21	Cr	(0), 13/12	1
3886.83	Cr	(0), 3/2	3	4325.16	Cr	(0), 15/17	2
3894.07	Cr	(0), 3/2	3	4337.59	Cr	(1, 0), 3, 2, 1/2	22
3902.95	Cr	(0), 3/2	3	4339.44	Cr	(2, 1, 0), 7, 6, 5, 4, 3/4	3
3908.77	Cr	(0), 3/2	3	4351.06	Cr	(3), 3, 0/2	1
3916.27	Cr	(0), 3/2	3	4359.64	Cr	(2, 1), 4, 3, 2, 1/2	22
3919.22	Cr	(0), 3/2	3	4363.13	Cr	(0), 5/6	3
3921.06	Cr	(0), 3/2	3	4371.30	Cr	(3, 2, 1, 0), 8, 7, 6, 5, 4, 3/4	2
3928.66	Cr	(0), 3/2	3	4375.36	Cr	(0), 12/13	2
3941.52	Cr	(0), 3/2	3	4376.81	Cr	(0), 13/11	2
3963.74	Cr	(0), 7/6	1	4391.77	Cr	(2, 1, 0), 5, 4, 3, 2, 1/2	8
3983.88	Cr	(0), 9/11	1	4403.47	Cr	(0), 2/2	4
3990.00	Cr	(0), 8/7	1	4432.20	Cr	(0), 3/2	4
3991.15	Cr	(0), 5/8	1	4458.53	Cr	(0), 12/11	1
3992.87	Cr	(0), 28/17	1	4482.89	Cr	(1), 4, 3/2	4
4001.46	Cr	(0), 22/21	1	4488.04	Cr	(0), 13/10	4
4003.33	Cr	(0), 20/21	1	4492.32	Cr	(1, 0), 4, 3, 2/2	1
4012.52	Cr	(0), 20/21	1	4496.86	Cr	(2, 1, 0), 7, 6, 5, 4, 3/3	23
4039.10	Cr	(0), 9/8	1	4498.73	Cr	(0), 31/21	12
4048.78	Cr	(0), 22/21	2	4501.28	Ti	(0), 9/10	1
4058.81	Cr	(0), 5/6	1	4506.80	Cr	(0), 20/21	3
4065.73	Cr	(0), 13/12	1	4511.92	Cr	(2), 7, 2/4	19
4077.69	Cr	(1), 5, 3, 1/3	1	4512.74	Ti	(0), 3/2	1
4121.29	Cr	(0), 2/2	1	4518.03	Ti	(0), 3/2	1
4121.85	Cr	(0), 2/2	1	4526.48	Cr	(0), 21/16	3
4123.40	Cr	(0), 7/6	1	4533.25	Ti	(0), 4/3	1
4126.54	Cr	(0), 4/3	1	4542.62	Cr	(0), 11/10	1
4142.15	Cr	(0), 5/4	1	4549.64	Ti	(0), 22/21	1
4161.45	Cr	(0), 13/12	3	4552.46	Ti	(0), 3/2	1
4163.65	Cr	(0), 7/6	1	4554.04	Ba	(1), 5, 3/3	10
4165.55	Cr	(0), 2/2	2	4555.49	Ti	(0), 3/2	1
4172.80	Cr	(0), 13/12	1	4556.16	Cr	(3), ?/5	2
4179.28	Cr	(0), 15/16	2	4563.77	Ti	(0), 4/5	2
4191.29	Cr	(0), 13/14	1	4564.18	Cr	(0), 2/2	2
4206.90	Cr	(0), 2/2	2	4569.62	Cr	(11), 26/15	1
4240.69	Cr	(0), 8/7	2	4571.98	Ti	(0), 20/21	1

TABLE I—*Continued*

4580.07	Cr	(1, 0), 5, 4, 3/2	36	4722.16	Zn	(1), 4, 3/2	11
4591.42	Cr	(2, 0), 13, 11, 9/6	4	4727.16	Cr	(0), 7/6	2
4595.58	Cr	(0), 2/2	3	4729.74	Cr	(0), 9/8	1
4613.33	Cr	(0), 5/2	23	4737.34	Cr	(0), 8/7	2
4616.11	Cr	(4, 2), 13, 11, 9, 7/6	22	4752.12	Cr	(0), 2/2	5
4619.54	Cr	(0), 31/21	2	4756.13	Cr	(0), 13/12	2
4626.16	Cr	(2), 5, 3/2	37	4758.13	Ti	(0), 2/2	1
4651.32	Cr	(2, 0), 5, 3, 1/2	35	4759.25	Ti	(0), 8/7	1
4652.21	Cr	(12, 6, 0), 38, 32, 26, 20, 14/17	12	4764.32	Cr	(0), 22/21	1
4654.74	Cr	(0), 6/2	4	4792.52	Cr	(6, 3, 0), 16, 13, 10/10	1
4656.47	Ti	(0), 9/11	1	4799.90	Cd	(1), 4, 3/2	1
4667.59	Ti	(0), 2/2	1	4801.03	Cr	(0), 4/3	1
4678.17	Cd	(0), 4/2	1	4810.55	Zn	(1, 0), 4, 3, 2/2	11
4680.17	Zn	(0), 4/2	11	4824.11	Cr	(0), 15/11	2
4681.91	Ti	(0), 16/15	1	4848.25	Cr	(0), 5/4	2
4589.40	Cr	(2, 1, 0), 5, 4, 3, 2, 1/2	5	4856.02	Ti	(0), 22/21	1
4693.96	Cr	(0), 2/2	6	4870.81	Cr	(0), 5/7	1
4700.62	Cr	(1, 0), 3/2	2				

To illustrate the meaning of the fractions the chromium line λ 4580.07 is drawn to scale in Figure 1. Lines at the top and bottom of the diagram indicate the position of the line in the absence of the magnetic field. The *p*-components are shown separated from the *n*-components just as they appear upon the photographs described below.

Table I also shows the number of plates upon which each line was measured. It will be seen that, in the main, chromium lines having small denominators were used between the wave-lengths λ 3883 and λ 4752. The complex types having six, twelve, or more components, when suitable for measurement, are of great usefulness on account of the checks they afford on the value of *a* for a given plate, and also because they increase the number of separate values
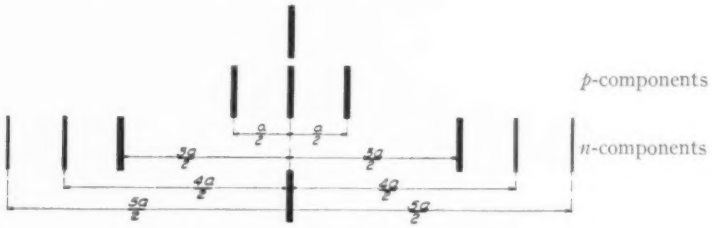
FIG. 1.—Zeeman effect for Cr λ 4580

obtainable. Triple lines whose separation is well known, but different from the normal interval, are just as useful for the determination of *a* as those which are normal. An example of this is λ 4613 of chromium, which is an excellent triple line whose separation is $5a/2$. Such lines are of more value than those having the normal separation because the percentage error in the value of *a* derived from them is less.

With regard to the lines in Table I to which are assigned fractions having large denominators, e.g., 13, 17, 21, it may be said that the extended investigation of the Zeeman effect for iron, chromium, and vanadium referred to above has furnished clear evidence for the existence of these denominators, which will be published as soon as possible. Upon the basis of 1177 lines of iron, 1123 of chromium, and 1462 of vanadium, for which the Zeeman effect has been measured, it becomes comparatively easy to ascertain the reality of even

such large denominators as 21. It will be observed, however, from an inspection of Table I, that these large denominators play a minor part in the present investigation.

In selecting lines for measurements of a the following considerations were kept in view: definiteness of the fractional system pertaining to the line, sharpness of the components, and absolute amount of their separation and number of measurable lines occurring on a given photograph. It is not feasible to use the fainter lines on account of circumstances relating to exposure time, heating of spark terminals, and the desirability of keeping the field strength as high as possible. The blue and violet region of the chromium spectrum fulfils the conditions satisfactorily.

In combining the separate values of a for a given photograph, weights were assigned depending on the ease of measurement and the absolute separation of the components. The probable error of the weighted means thus derived shows that when 40 or 50 separate measurements of a are combined, the result for the plate has an uncertainty of about 1 part in 600.

The measured displacements of the magnetically separated components were reduced to angstroms by means of a series of curves obtained from measurements on iron lines of known wavelength. The same spectrographic adjustments were used for these lines as in the general study of the Zeeman effect, the source being an iron arc.

The magnetic field was supplied by a large oil- and water-cooled magnet designed by Weiss, which has already been described.[1] Examination of the performance of the magnet under actual working conditions showed that for the portion of the magnetization curve employed in this work no appreciable error is introduced by breaking the magnetizing current during an exposure, or between the exposure and the measurement of the magnetic field-strength. The control of the magnetizing current was accomplished by means of a fine adjustment of the voltage of the generator which supplied the power, the current being measured and kept constant throughout each exposure by means of a sensitive ammeter. For different

[1] Annual Report, Mt. Wilson Observatory, *Year Book Carnegie Institution of Washington*, No. 11, 1912, p. 202.

observations the current ranged between 35 and 70 amperes, while the air gap in the magnetic circuit was from 7 to 12 mm. The lowest field-strength used was 24,290 gausses, the highest 31,880 gausses. The magnet is so mounted that the beam of light may traverse the field either at right angles or parallel to the lines of force. Both arrangements were used, but the former more frequently, since it has the advantage of giving all the components into which a line may be divided. It has, however, the disadvantage of reducing the n-components to one-half the intensity which they have when viewed along the lines of magnetic force. It was customary, when working with light perpendicular to the field, to separate the p- and n-components along the length of the slit by placing a nicol prism before the slit with an orientation such as to transmit one group of components and suppress the other. One-half the nicol and a corresponding segment of the slit were then covered by a half-wave plate of mica, with its axis inclined 45° to the slit. The combination of half-wave plate and nicol transmitted the group of components extinguished by the nicol alone, so that both groups could be simultaneously photographed. In the case of light parallel to the field no analyzer was required, since the n-components alone were visible and there was no occasion for distinguishing between the two states of circular polarization presented.

Measurements of the field-intensity were made by removing from the air gap a very small test coil which was connected in series with a long-period ballistic galvanometer, a manganin resistance of 20,000 ohms, and the secondary circuit of a mutual inductance. Galvanometer deflections of opposite sign were obtained by inverting the test coil between successive withdrawals. This method was found preferable to that of inverting the coil in the field, on account of the smallness of the air gap in which the coil had to be placed. The series of deflections thus secured was intermingled with another series obtained by reversing known currents in the primary of the mutual inductance while the test coil was stationary outside of the field of the magnet. The current which would give the same deflection as that produced by the test coil could then be found by interpolation.

Let I represent this current expressed in c.g.s. units, M, the mutual inductance in centimeters, and f, the magnetic area of the

test coil in square centimeters; the field-strength, H, is then given by the expression

$$H = \frac{2MI}{f} \text{ gausses.}$$

The factor 2 is required by the fact that I is reversed in direction, while the test coil is not inverted but merely withdrawn from the field. Since nearly all the resistance in the ballistic galvanometer circuit was of manganin, no correction was needed for the small changes of temperature which occurred during a series of measurements. This high resistance also made unnecessary any correction for damping for such a range of deflections as was used.

The mutual inductance was designed for these measurements. It was sent to the Bureau of Standards for certification, where its value was found to be 3.918 millihenrys. Its construction is such as to insure a high degree of permanence in this value. The two circuits were wound simultaneously with their respective wires side by side, so that the mutual inductance is nearly equal to the self-inductance of each circuit. The spool on which the wire is wound is made of seasoned mahogany. The terminals are small and as far from the region of concentrated magnetic field as possible. When in use the coil was supported upon dry wood well away from metallic objects and far beyond the influence of the large magnet whose field-strength was being measured.

A second mutual inductance whose value was obtained by comparison with that described above, and by independent measurements in terms of a known capacity and known resistance according to the Carey-Foster method, was used for part of the observations. It is of similar construction and has a mutual inductance of 17.25 millihenrys.

Two test coils were used in the direct observations while five other auxiliary coils were employed in determining the magnetic areas. The two principal coils have magnetic areas of 14.112 sq. cm and 7.434 sq. cm, respectively. They are wound on ivory spools and have several layers of No. 40 enameled copper wire. The dimensions of the spools and of each layer of wire were measured during their construction by mounting them under the microscope of a measuring machine. From the number of turns in each layer and the mean diameters the effective areas could be calculated.

The screw of the measuring machine was standardized by means of a precision scale ruled upon glass, calibrated and certified by the Bureau of Standards. Independent determinations of the areas were then made by intercomparing these coils with the five auxiliary coils, which were all single layer coils whose areas could be calculated with higher accuracy than is possible with multiple layer coils. In order to make the areas of the single-layer auxiliary coils nearly equal to those of the multiple-layer coils, which is desirable for reducing the errors of comparison, their diameters were increased. On account of their large volume the single-layer coils were not suited to direct measures on concentrated fields.

For comparing the areas of different types of coils the following plan was adopted. The two coils to be compared were temporarily fastened together and connected in series with each other and with the ballistic galvanometer and a suitable high resistance. They were then placed in a uniform magnetic field of sufficient intensity, obtained from the Weiss magnet by using an air gap of 30 mm between circular pole faces 50 mm in diameter. The coils were placed in position near the center of the field and, after quieting the galvanometer, were quickly withdrawn and the ballistic deflection was read. They were then inverted and a corresponding deflection of opposite sign was observed. After recording a series of such deflections, the terminals of one of the two test coils were reversed and the series was repeated. The amplitude of the deflections depends on the direction of the respective windings of the two coils, and will be proportional to either the sum or the difference of their magnetic areas. Simple combinations of the observations, therefore, give the ratio of the areas of the two coils.

This method was extended by connecting three test coils in series instead of two, each one of the coils in turn being kept outside of the magnetic field while the other two were used as described above. The ratio of a given pair of areas was in this way measured directly as well as indirectly, thus affording a useful check on the accuracy. The calculated and observed areas agree within the range indicated by their probable errors. It is thought that the mean area of the two coils used for the final measurements may be relied upon to 1 part in 1500.

The constants M and f being evaluated, consider now the measurement of the variable quantity, I, the current which is reversed in the primary circuit of the mutual inductance. At the beginning of the investigation this current was measured simultaneously by two separate instruments, a high-grade direct-reading ammeter, and a galvanometer with low-resistance shunt. These instruments were standardized by comparison with a silver voltameter, observing the usual precautions required in this operation.

A more satisfactory method was soon adopted, however, whereby the galvanometer was used only as a null reading indicator, thus avoiding the necessity for frequent standardization and at the same time securing higher accuracy. The current to be measured was passed through a standard 10-ohm resistance coil shunted by an adjustable resistance whose value was accurately known. The fall of potential across this combination was measured by opposing it to that of a Weston Standard cell with galvanometer in series, as in the ordinary potentiometer method. For each chosen value of the adjustable resistance the current flowing through the primary of the mutual inductance was varied until the galvanometer in series with the standard cell showed no deflection, whereupon the direction of the current through the mutual inductance was reversed by a quick-acting switch and the ballistic deflection was read. From the known values of the resistance and of the voltage of the standard cell the corresponding values of the current were derived by Ohm's law.

Two standard cells of the Weston type were used. One has for its electrolyte a solution of cadmium sulphate which is saturated at $+4°$ C., while the other has an excess of cadmium sulphate crystals. The former cell was standardized at the Physikalisch-Technische Reichsanstalt and later both cells were compared with a new Weston cell having a manufacturer's certificate. The mean e.m.f. of the two cells was 1.0183 international volts. The temperature coefficients are negligible over the range of temperatures at which the observations were made.

The ballistic galvanometer has a resistance of about 2100 ohms and a period of 16 seconds. It is of the moving-coil type and was used at a scale distance of 4 meters with an observing telescope

giving a magnification of 36 diameters. Under these conditions the ballistic sensitivity of the galvanometer was 6×10^{-10} coulombs per millimeter of deflection. The galvanometer used in the potentiometer circuit was sufficiently sensitive to show variations amounting to a few parts in 100,000 in the current passing through the standard resistance.

The standard 10-ohm coil used in this work was constructed by Otto Wolff according to the specifications developed by the Bureau of Standards.[1] No correction was required for temperature. The shunt in parallel with the standard coil was taken from a box of high-quality manganin resistances which was calibrated in terms of the 10-ohm standard.

An examination of the combined effect of the uncertainties in the e.m.f. of the standard cell and in the resistance shows that the probable error of the current can hardly exceed 1 part in 5000.

It is well known that with magnetic fields such as those used in this work the intensity varies considerably from point to point within the air gap. This effect may easily be shown either by a suitable exploring device such as a small bismuth spiral, or by means of the Zeeman effect itself. In order to avoid as far as possible the effects of this non-uniformity of the field, the spectrograph slit was placed at right angles to the direction of the magnetic field and was kept rather short. As a result the spectral lines appeared straight on the photographs. When the field-strength is measured with a test coil, however, the volume occupied by the coil makes a certain amount of integration inevitable. In order to correct for the difference between the field which was effective for the photographs and that which was measured, it was necessary to map the intensity from pole to pole across the air gap over an area equivalent to the cross-section of the test coil. The bismuth spiral, although not well adapted to precise absolute determinations of magnetic intensity, is nearly ideal for such relative measurements as are required for this purpose. It was mounted with its plane parallel to the pole faces of the magnet and was moved by successive small steps across the air gap, its resistance being read at each point.

[1] *Bulletin of the Bureau of Standards*, **5**, 413, 1908–1909.

The resulting curves of field-strength as a function of position in the air gap were then plotted and mechanically integrated over the volume occupied by the test coil in order to find the desired correction. This quantity was generally 175 or 200 gausses.

Since, as stated above,

$$\frac{e}{m}=\frac{4\pi aV}{H},$$

and

$$H=\frac{2MI}{f},$$

the working equation is

$$\frac{e}{m}=\frac{2\pi aVf}{MI}.$$

To determine the combined effect of the uncertainties of the various factors on the resulting value of e/m, the following values, which closely represent those actually occurring in the observations, were assigned arbitrarily. The appended quantities are probable errors, for a single photograph in the case of a, and for a single complete determination of the field-strength in the case of I.

$a=1.230\pm0.002$ ..... cm^{-1}
$v=(2.9978\pm0.0003)10^{10}$ ..... cm/sec
$f=14.112\pm0.0093$ ..... cm^2
$M=(3.918\pm0.0005)10^{6}$ ..... cm
$I=(0.4750\pm0.0001)10^{-1}$ ..... c.g.s.

Let $R=$probable error of e/m. Then, by the usual equation for the propagation of errors,

$$R^2=\left\{\frac{2\pi vf}{MI}\right\}^2 4\times10^{-6}+\left\{\frac{2\pi af}{MI}\right\}^2 9\times10^{12}+\left\{\frac{2\pi av}{MI}\right\}^2 8.6\times10^{-5}$$
$$+\left\{\frac{2\pi avf}{IM^2}\right\}^2 25\times10^{4}+\left\{\frac{2\pi avf}{MI^2}\right\}^2 10^{-10}.$$

Evaluating the successive terms in this expression, we find

$$R^2=(8.16+0.03+1.33+0.05+0.14)\times10^{8}=9.71\times10^{8}$$

and

$$R=\pm3\times10^{4}=\pm0.003\times10^{7}$$

Since the value of e/m is approximately 1.76×10^7, the percentage probable error of the result derived from a single photograph is 1 part in 600, which is the same as that for the corresponding determination of a. The uncertainties of the other factors in the equation are negligible except in the case of the magnetic area of the test coil, which contributes, however, only one-seventh of R^2.

Table II shows the results of all the measurements made. The weight assigned to each value of e/m depends on the sum of the weights for the individual observations of a on the corresponding photograph. Among the first 12 plates, the weights were in some cases reduced from 2 to 1 because of the less accurate method used

TABLE II

Plate No.	a	H	e/m	Wt.	Plate No.	a	H	e/m	Wt.
1.....	1.352	29030	1.7543	1	26.....	1.486	31810	1.7598	4
2.....	1.346	29030	1.7465	1	27.....	1.243	26500	1.7670	4
3.....	1.300	29030	1.7647	1	28.....	1.234	26500	1.7542	4
4.....	1.274	27670	1.7344	1	29.....	1.228	26500	1.7457	2
5.....	1.287	27670	1.7521	1	30.....	1.229	26500	1.7471	3
6.....	1.284	26950	1.7946	1	31.....	1.233	26500	1.7528	4
7.....	1.281	26950	1.7904	1	32.....	1.233	26500	1.7528	3
8.....	1.385	29360	1.7709	1	33.....	1.236	26500	1.7571	2
9.....	1.423	30300	1.7690	1	34.....	1.241	26500	1.7642	2
10.....	1.411	30300	1.7542	1	35.....	1.241	26500	1.7642	2
11.....	1.306	27660	1.7786	1	36.....	1.143	24290	1.7727	4
12.....	1.341	28320	1.7837	1	37.....	1.145	24290	1.7758	4
13.....	1.485	31810	1.7586	5	38.....	1.138	24290	1.7649	4
14.....	1.494	31810	1.7693	4	39.....	1.243	26500	1.7670	4
15.....	1.487	31810	1.7610	4	40.....	1.236	26500	1.7571	3
16.....	1.477	31810	1.7572	4	41.....	1.239	26500	1.7613	2
17.....	1.490	31810	1.7646	2	42.....	1.238	26500	1.7599	4
18.....	1.475	31810	1.7408	5	43.....	1.242	26500	1.7656	4
19.....	1.498	31810	1.7740	5	44.....	1.242	26500	1.7656	2
20.....	1.404	31810	1.7338	4	45.....	1.235	26500	1.7556	3
21.....	1.485	31810	1.7586	5	46.....	1.236	26500	1.7571	3
22.....	1.483	31810	1.7563	5	47.....	1.232	26500	1.7514	3
23.....	1.490	31810	1.7646	5	48.....	1.242	26500	1.7656	2
24.....	1.484	31810	1.7575	5	49.....	1.248	26500	1.7741	2
25.....	1.490	31810	1.7646	2					

for measuring the current in the mutual inductance. The most probable value of e/m is the weighted mean of all the determinations. This is found to be

$$\frac{e}{m} = 1.761 \times 10^7$$

with a probable error of 1 part in 1800.

It would be manifestly unfair to give each observation the same weight, on account of the variation in the amount of data involved, but it is interesting to note that, if they are combined in this manner,

the mean becomes 1.762×10^{7}. This indicates that within fairly wide limits no great importance attaches to the weight assigned.

As noted above, the probable error of the weighted mean is 1 part in 1800. From this and the preceding analysis of the uncertainty of e/m as dependent on the various factors involved, considered in connection with the amount of observational material, it appears reasonable to believe that the actual uncertainty in the final value of e/m is not more than 2 or 3 parts in 1800.

This conclusion is supported by the recent work of Birge[1] on the value of the Planck constant h. Using the value

$$e/m=1.761\times10^{7},$$

he finds

$$h=6.556\times10^{-27}$$

while the most probable value of h, which he derives by seven different methods, using revised data, is

$$h=6.557\times10^{-27}$$

Although it is thought that the value of e/m found here may properly claim considerable accuracy, further improvements now seem quite possible. Inasmuch as the chief difficulty lies in the determination of a, it is evident that the use of much higher resolving power would yield a marked gain. In addition, a more extended list of spectral lines is now available for which fractional separations are sufficiently well known to make them useful. Since the completion of the observations described here, a powerful solenoid[2] capable of furnishing fields of 35,000 gausses has been constructed in the instrument shop of the Observatory. With this instrument and the other improvements referred to, it is hoped that in the future our knowledge of e/m may be still further increased.

My grateful acknowledgments are due to Miss Lois M. Keener and several former members of the Computing Division for much assistance in the measurements and reductions involved in this work.

[1] *Nature*, **111**, 811, 1923; *Physical Review*, **14**, 361, 1919.

[2] Annual Report, Mt. Wilson Observatory, *Year Book Carnegie Institution of Washington*, No. 18, 1919, pp. 220, 251; *ibid.*, No. 19, 1920, p. 256; *ibid.*, No. 20, 1921, p. 285.

MOUNT WILSON OBSERVATORY
May 1923

PHOTOMETRIC STUDIES OF SZ HERCULIS AND RS VULPECULAE

By RAYMOND S. DUGAN

ABSTRACT

Studies of the eclipsing variables SZ Herculis and RS Vulpeculae with the polarizing photometer have been carried out and the *mean light curves* of these stars have been drawn and discussed. The probable error of a single observation with the comparison star $+33°2925$ is $\pm 0^{M}026$. *New light elements* are given, also *tables* of normals and plotted light curves. For both stars the uniform curve represents the observations better than the darkened. The spectroscopic observations aided in the selection of the correct interpretation of the light curve of RS Vulpeculae. Orbital elements and other data are summarized.

The variability of star $+33°2931$, one of the comparison stars of SZ Herculis, is apparent from the distribution of the residuals.

SZ Herculis (1900, $\alpha = 17^{h}36.^{M}0$, $\delta = +33°1'$, $10.^{M}0$).—Observation of this star was begun in 1915. At that time sufficient observations had not been made during primary minimum to define this part of the light curve satisfactorily. Very few observations had been made when the star was free from eclipse and nothing was known concerning the secondary minimum. The star $A = +33°2931$ proved to be variable over a small range. The observations are not sufficient to determine the nature of the variation. The other comparison star, $B = +33°2925$, showed no variation. The distribution of the residuals of the B–V observations is quite normal, while the non-accidental occurrence of the A–V observations is apparent. These conclusions are represented also by the size of the probable errors,

$\pm 0.^{M}026$ for a B–V observation,
$\pm 0.^{M}042$ for an A–V observation.

The B–V observations are not sufficiently numerous to independently determine all parts of the curve. In order to make use of the A–V observations, a preliminary solution was made of the B–V observations and values for ellipticity and reflection assumed on the basis of the values of the relative dimensions thus secured. The secondary is well defined by the B–V observations. Residuals of the A–V observations from this provisional theoretical curve

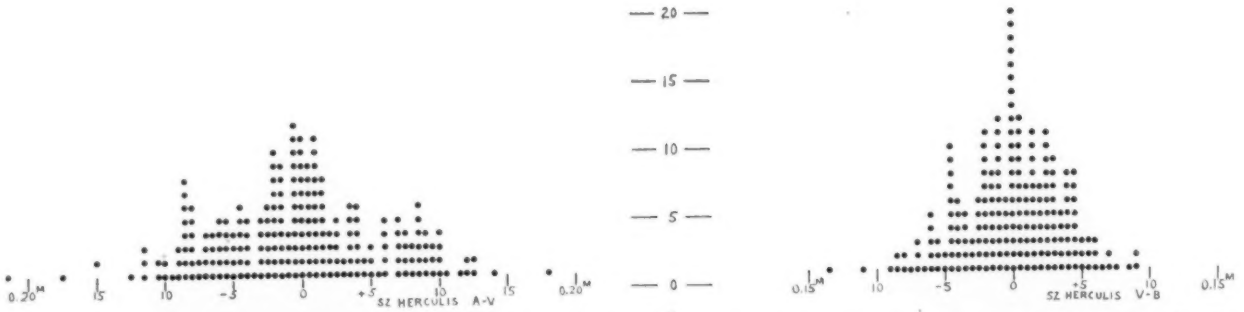

FIG. 1.—Distribution of residuals of observations of SZ Herculis. The V-B residuals are apparently accidental, while the distribution of the A-V residuals indicates that comparison star A is variable.

were read off and their average value on each night applied to the A–V observations of that night. The final values of the ellipticity and reflection coefficients are somewhat affected by this procedure and should not be used in any general discussion of these effects.

The newly determined heliocentric elements are

$$\text{Min.} = \text{J.D. } 2418495.406 + 0^{d}.8180972\ E.$$

The letter following the current number in the table of normals indicates the comparison star used. The sum of the weights of the

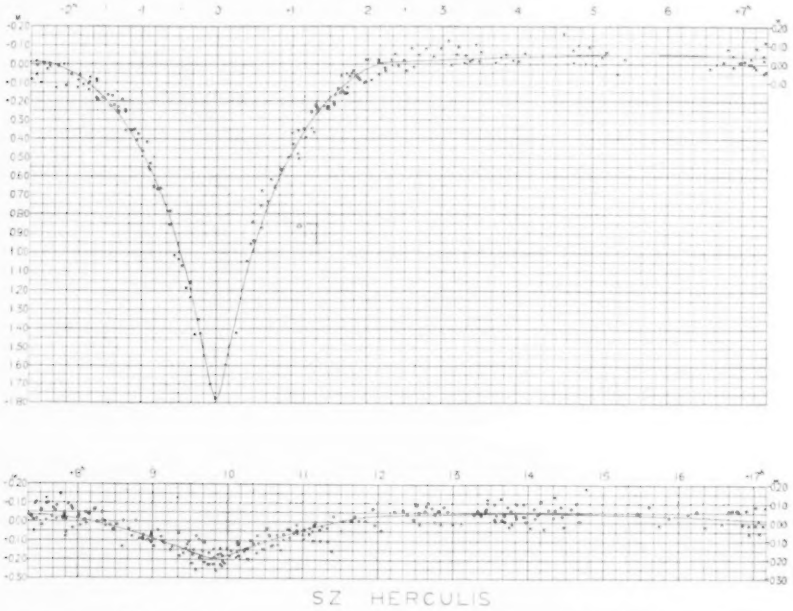

FIG. 2.—Mean light curve of SZ Herculis. Single observations plotted. Theoretical "uniform" curve drawn out.

observations in a normal is reduced by a factor which is unity when no two observations in the normal were made on the same night and is 0.5 when all the observations were made on one night. O–C is the deviation of the observed normal from the final computed curve.

In Figure 2 the individual observations are plotted and the accepted theoretical curve drawn out.

The solution assumes stars presenting uniformly illuminated disks. No satisfactory "totally darkened" solution was found.

RS Vulpeculae ($\alpha = 19^h13^m.4$, $\delta = +22°16'$, $7^m.0$, B8, B9).—This star was observed on 63 nights between October, 1919 and December, 1922. It was necessary to cover thoroughly the primary and the region of a possible secondary minimum and a few points between eclipses.

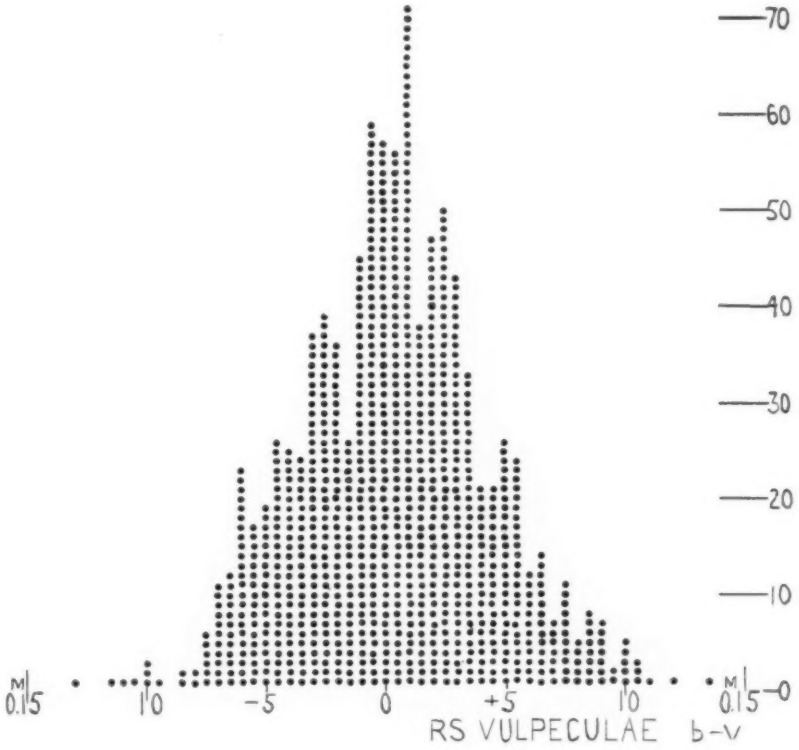

FIG. 3.—Distribution of residuals of observations of RS Vulpeculae. An apparently normal distribution giving no evidence of any variation of the comparison star.

New light elements were adopted as follows:

$$\text{Min.} = \text{J.D. } 2420606.6226 + 4^d.477666\ E.$$

Comparison star $b = +22°3644$ was used throughout the series and gave no indication of being itself a variable. The probable error of a single observation is $\pm 0^M.026$.

The curve drawn through the normals is the final curve of the uniform solution of partial eclipse. The variation in brightness when the system is free of eclipse and during secondary minimum, in the lower part of the figure, has been exaggerated by increasing the magnitude scale and compressing the time scale. The two minima are drawn the same length.

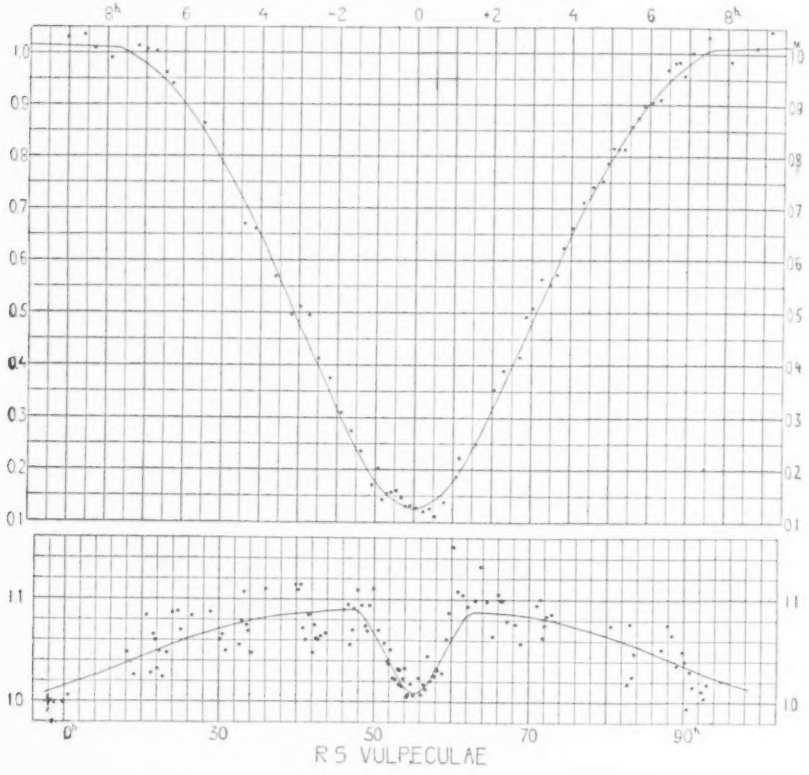

FIG. 4.—Mean light curve of RS Vulpeculae. Normals plotted. Theoretical "uniform" curve drawn out.

The ellipticity and reflection coefficients are well determined by the observations and the secondary minimum is clearly outlined.

The change in brightness at bottom of primary minimum is very gradual. One would hardly quarrel with the assertion that the light is constant for an hour or so. The interpretation of primary minimum as a total eclipse is not consistent with the value of the

ratio of diameters and the depth of secondary minimum. Plaskett's statement that the spectral lines of the faint component are visible and about one-fourth as strong as those of the bright component makes it impossible to accept the interpretation of primary minimum as an annular eclipse. If it were not for the spectrographic evidence one might be almost content with the representation of the

SUMMARY OF RESULTS

	SZ Herculis	RS Vulpeculae
Nature of each eclipse	Partial	Partial
Duration of each eclipse	4^h30^m	15^h19^m
Magnitude at maximum	10.0	7.0
Magnitude at primary minimum	11.8	8.0
Magnitude at secondary minimum	10.2	7.1
Loss of light at primary, due to eclipse	1^M64	0^M97
Loss of light at secondary, due to eclipse	0^M22	0^M08
Displacement of secondary minimum	0	$+1\frac{1}{2}^h$
Inclination of orbit plane	88°.4	79°.0
Ratio of radii of two stars	0.960	0.765
Ratio of axes of each star	0.965	0.984
Fraction of surface of brighter star covered at primary minimum	0.977	0.636
Difference in intensity of sides of fainter star	0.030	0.078
Light of brighter star	0.800	0.804
Light of bright side of faint star	0.200	0.196
Ratio of surface brightness	4.0	4.1
Max. radius of brighter star	0.3295*	0.2045*
	1.5⊙†	4.27⊙
Max. radius of fainter star	0.3432*	0.2674*
	1.6⊙†	5.58⊙
Distance of centers	4.6⊙†	20.86⊙
Mass of brighter star	1.4‡	4.59⊙
Mass of fainter star	0.6‡	1.44⊙
Density of brighter star	0.38	0.060
Density of fainter star	0.16	0.008

* Distance of centers is unity.
† Mass of each component assumed equal to mass of sun.
‡ Assumed.

observations on this hypothesis. There remains the case of partial eclipse. The theoretical curve fits the observations very satisfactorily. Again there is no evidence of darkening at the limb.

The secondary is displaced toward the following primary about $1\frac{1}{2}$ hours. Plaskett's spectroscopic orbit, of which the eccentricity is but 0.05, would require a shift of about two hours in the opposite direction. Otherwise the light curve is sensibly symmetrical.

TABLE OF NORMALS. SZ HERCULIS

Norm. No.	Phase	Wt.	V–B	O–C
1B	− 2h20m63	2.5	0M000	+0M012
1A	20.30	3.5	+0.077	+ 89
2A	2 1.20	4	+0.047	+ 24
3A	1 48.90	4	+0.097	+ 24
4A	38.90	4	+0.131	+ 2
2B	33.50	1.5	+0.122	− 39
3B	20.30	3.5	+0.248	− 11
5A	19.88	3	+0.205	− 58
4B	6.50	2.5	+0.379	− 20
6A	1 3.50	2.5	+0.374	− 59
5B	0 53.75	2.5	+0.563	+ 4
7A	49.33	2	+0.618	+ 1
8A	35.38	1.5	+0.892	+ 32
6B	30.50	1	+1.043	+ 78
9A	22.00	1.5	+1.168	+ 13
7B	17.25	1	+1.293	0
10A	14.00	1.5	+1.426	+ 36
11A	− 0 3.13	1.5	+1.726	− 14
12A	+ 0 11.75	.5	+1.508	+ 48
8B	29.75	3	+0.875	− 102
13A	35.50	1.5	+0.775	− 79
9B	0 43.63	2	+0.708	− 6
14A	1 1.50	3	+0.479	+ 23
10B	7.63	2	+0.355	− 29
15A	13.38	3.5	+0.341	+ 20
16A	24.20	4	+0.233	+ 6
11B	26.83	1.5	+0.208	− 1
17A	36.40	4.5	+0.172	+ 28
18A	44.60	4.5	+0.096	+ 1
12B	49.10	2.5	+0.080	+ 12
19A	1 56.40	4	+0.039	+ 3
20A	2 11.33	3.5	+0.001	+ 5
13B	17.70	3	+0.016	+ 28
14B	35.00	3.5	−0.021	− 5
15B	2 59.40	2.5	−0.085	− 63
21A	3 19.33	3	−0.039	− 12
16B	32.30	3.5	−0.025	+ 5
17B	3 58.63	2	−0.035	0
22A	4 47.05	4	−0.070	− 26
23A	5 14.28	1.5	−0.009	+ 37
24A	6 43.90	3	−0.022	+ 21
25A	6 59.60	4	−0.004	+ 36
26A	7 10.80	4.5	−0.027	+ 12
18B	19.50	3.5	+0.015	+ 53
27A	+ 7 21.10	4.5	−0.029	+0.007

TABLE OF NORMALS. SZ HERCULIS—*Continued*

Norm. No.	Phase	Wt.	V–B	O–C
28A.....	+ 7^{h}29^{m}80	4	−0^{M}065	−0^{M}031
29A.....	39.20	4.5	−0.065	− 33
19B.....	43.39	3.5	−0.073	− 45
30A.....	49.50	5	−0.023	+ 3
20B.....	7 54.80	4.5	−0.020	0
21B.....	8 3.50	4	−0.023	− 15
22B.....	11.80	4.5	−0.033	− 33
23B.....	22.00	4.5	−0.004	− 18
24B.....	38.40	4	+0.042	− 3
31A.....	42.21	2	+0.060	+ 8
32A.....	57.93	2	+0.075	− 7
25B.....	8 59.70	4	+0.090	+ 3
26B.....	9 20.00	4	+0.126	− 7
33A.....	21.00	2	+0.137	+ 2
27B.....	31.70	5	+0.184	+ 23
28B.....	37.70	5	+0.142	− 35
29B.....	43.50	5	+0.188	0
30B.....	50.33	4.5	+0.194	− 1
31B.....	55.50	5	+0.195	+ 9
34A.....	9 56.83	1.5	+0.198	+ 14
32B.....	10 3.50	4.5	+0.165	− 4
35A.....	10.80	4.5	+0.154	+ 4
33B.....	14.20	4	+0.132	− 9
36A.....	21.10	4	+0.131	+ 4
34B.....	32.80	4	+0.113	+ 13
37A.....	34.72	3	+0.077	− 17
35B.....	45.80	4	+0.087	+ 14
36B.....	10 55.60	4.5	+0.050	0
37B.....	11 4.90	4	+0.061	+ 28
38A.....	7.28	3.5	+0.038	+ 8
39A.....	21.17	4	+0.053	+ 45
38B.....	33.00	2.5	+0.015	+ 23
39B.....	50.83	2	+0.002	+ 29
40A.....	11 59.25	1	−0.013	+ 20
40B.....	12 26.88	2	−0.066	− 28
41A.....	12 31.00	3	−0.039	− 1
42A.....	44.50	4	−0.052	− 12
43A.....	12 57.30	3.5	−0.022	+ 19
41B.....	13 24.10	4	−0.065	− 21
44A.....	28.50	3	−0.048	− 3
42B.....	13 35.70	4.5	−0.050	− 4
43B.....	43.60	4.5	−0.031	+ 15
44B.....	52.60	4	−0.049	− 3
45A.....	13 59.60	3.5	−0.028	+ 18
46A.....	14 16.70	3.5	−0.068	− 22
45B.....	22.33	4	−0.023	+ 22
47A.....	14 35.13	2.5	−0.084	− 39
48A.....	15 40.25	2	−0.037	− 2
46B.....	+16 54.00	5	−0.002	+0.014

TABLE OF NORMALS. RS VULPECULAE

Norm. No.	Phase	Wt.	b–v	O–C
1.....	− 9^h 1^{m}3	2.5	1^{M}030	+0^{M}017
2.....	8 34.7	4.0	1.036	+ 24
3.....	8 19.7	3.5	1.009	− 3
4.....	7 53.7	2.5	0.991	− 19
5.....	7 11.8	3.5	1.013	+ 24
6.....	6 58.5	3.5	1.006	+ 28
7.....	6 43.7	3.0	1.003	+ 41
8.....	6 28.9	3.5	0.962	+ 17
9.....	6 18.1	3.5	0.940	+ 9
10.....	5 29.8	3.0	0.863	+ 14
11.....	4 25.2	3.5	0.669	− 41
12.....	4 8.0	3.0	0.661	− 8
13.....	3 36.9	2.5	0.570	− 13
14.....	3 11.5	4.0	0.497	− 14
15.....	2 58.6	4.0	0.512	+ 38
16.....	2 44.0	4.0	0.496	+ 59
17.....	2 29.1	4.0	0.413	+ 16
18.....	2 11.9	3.5	0.375	+ 25
19.....	1 53.9	3.5	0.310	+ 10
20.....	1 37.8	3.5	0.275	+ 16
21.....	1 22.1	3.0	0.236	+ 13
22.....	1 4.5	4.0	0.172	− 13
23.....	0 55.7	4.5	0.203	+ 33
24.....	0 49.5	4.5	0.143	− 19
25.....	0 42.1	4.5	0.154	+ 2
26.....	0 35.3	4.0	0.157	+ 10
27.....	0 27.8	5.0	0.161	+ 22
28.....	0 19.9	4.5	0.147	+ 12
29.....	0 13.3	5.0	0.130	− 1
30.....	− 0 5.9	4.0	0.132	+ 5
31.....	+ 0 3.9	4.0	0.127	0
32.....	0 14.7	4.5	0.121	− 11
33.....	0 24.4	4.0	0.126	− 12
34.....	0 32.8	4.5	0.112	− 33
35.....	0 46.4	3.5	0.138	− 20
36.....	1 5.3	3.0	0.187	− 1
37.....	1 35.6	2.5	0.251	− 2
38.....	2 2.7	3.5	0.353	+ 28
39.....	2 17.1	3.0	0.390	+ 27
40.....	2 41.9	4.0	0.416	− 14
41.....	2 51.4	4.0	0.492	+ 35
42.....	3 1.6	4.0	0.508	+ 24
43.....	3 15.0	3.5	0.564	+ 42
44.....	3 28.5	4.0	0.553	− 5
45.....	+ 3 38.3	4.0	0.573	−0.014

TABLE OF NORMALS. RS VULPECULAE—*Continued*

Norm. No.	Phase	Wt.	b–v	O–C
46.....	+ $3^h48^m.7$	4.0	$0^M.624$	$+0^M.008$
47.....	4 1.3	3.5	0.662	+ 10
48.....	4 18.2	3.5	0.712	+ 16
49.....	4 33.0	4.0	0.742	+ 13
50.....	4 48.6	4.5	0.753	− 12
51.....	4 56.5	4.5	0.788	+ 6
52.....	5 5.1	4.5	0.816	+ 15
53.....	5 13.0	4.5	0.814	− 3
54.....	5 21.2	4.0	0.814	− 20
55.....	5 33.4	4.0	0.858	+ 1
56.....	5 42.2	4.0	0.874	0
57.....	5 51.4	4.5	0.898	+ 8
58.....	6 3.3	4.0	0.905	− 4
59.....	6 16.8	4.0	0.910	− 19
60.....	6 27.9	4.5	0.968	+ 24
61.....	6 38.2	4.5	0.982	+ 25
62.....	6 45.3	4.5	0.983	+ 18
63.....	6 53.1	4.0	0.957	− 16
64.....	7 5.5	4.0	1.002	+ 17
65.....	7 30.2	2.5	1.032	+ 26
66.....	8 5.4	3.0	0.985	− 26
67.....	8 45.3	3.5	1.012	− 1
68.....	9 8.4	4.0	1.043	+ 30
69.....	9 48.6	3.0	0.999	− 16
70.....	10 28.7	3.0	1.006	− 11
71.....	18 6.3	3.0	1.048	+ 10
72.....	19 1.1	3.0	1.026	− 15
73.....	20 37.5	3.0	1.084	+ 38
74.....	21 12.2	3.5	1.028	− 20
75.....	21 30.8	4.0	1.066	+ 18
76.....	21 45.1	4.5	1.060	+ 11
77.....	22 1.8	4.5	1.032	− 18
78.....	22 18.5	4.0	1.050	− 2
79.....	22 44.8	3.5	1.024	− 29
80.....	23 18.6	2.5	1.048	− 7
81.....	23 59.9	2.5	1.087	+ 31
82.....	24 41.7	4.0	1.088	+ 30
83.....	25 4.2	4.0	1.070	+ 11
84.....	26 30.4	3.0	1.084	+ 21
85.....	28 57.3	3.0	1.088	+ 19
86.....	30 7.0	3.0	1.061	− 10
87.....	30 25.7	3.5	1.066	− 7
88.....	30 55.4	3.0	1.050	− 24
89.....	32 39.9	3.5	1.056	− 21
90.....	+32 57.3	3.5	1.079	+0.002

TABLE OF NORMALS. RS VULPECULAE—*Continued*

Norm. No.	Phase	Wt.	b–v	O–C
91	+33^{h}15^{m}5	3.5	1^{M}107	+0^{M}030
92	33 31.3	3.5	1.075	− 4
93	33 47.2	3.5	1.069	− 10
94	34 13.6	2.5	1.048	− 32
95	36 3.5	0.5	1.110	+ 28
96	39 55.6	3.5	1.114	+ 28
97	40 14.6	3.5	1.109	+ 23
98	40 33.5	3.5	1.114	+ 28
99	40 51.1	3.5	1.072	− 15
100	41 9.6	3.5	1.061	− 26
101	41 32.1	3.5	1.085	− 2
102	41 46.4	3.5	1.085	− 2
103	42 6.8	4.0	1.048	− 39
104	42 18.7	4.0	1.075	− 13
105	42 29.9	4.0	1.062	− 26
106	42 41.9	4.0	1.061	− 27
107	43 7.8	5.0	1.064	− 24
108	43 45.3	2.5	1.067	− 22
109	46 9.0	3.0	1.090	0
110	46 40.1	3.0	1.095	+ 5
111	46 56.9	3.5	1.056	− 34
112	47 11.8	3.5	1.070	− 20
113	47 26.1	3.0	1.091	+ 1
114	47 53.9	2.5	1.109	+ 20
115	48 23.5	2.5	1.094	+ 9
116	48 51.7	3.0	1.074	− 7
117	49 10.0	3.5	1.069	− 9
118	49 27.0	3.5	1.094	+ 20
119	49 59.1	2.5	1.110	+ 43
120	50 35.4	2.5	1.070	+ 12
121	51 21.4	4.0	1.057	+ 8
122	51 45.0	4.0	1.039	− 4
123	51 57.0	4.0	1.037	− 3
124	52 9.9	3.5	1.051	+ 14
125	52 23.9	3.5	1.023	− 10
126	52 43.5	3.0	1.022	− 6
127	53 1.4	4.0	1.032	+ 8
128	53 12.2	4.0	1.017	− 5
129	53 22.5	4.0	1.031	+ 11
130	53 33.6	4.0	1.015	− 3
131	53 47.1	4.0	1.022	+ 7
132	54 0.1	4.0	1.033	+ 20
133	54 13.1	3.5	1.005	− 7
134	54 24.6	3.5	1.007	− 4
135	+54 41.4	3.0	1.017	+0.008

TABLE OF NORMALS. RS VULPECULAE—*Continued*

Norm. No.	Phase	Wt.	b–v	O–C
136	+55^h 7^{m}6	2.0	1^{M}006	−0^{M}002
137	55 47.0	2.5	1.023	+ 13
138	56 6.7	3.5	1.007	− 4
139	56 23.7	3.5	1.017	+ 4
140	56 39.3	3.5	1.012	− 3
141	56 57.5	3.5	1.045	+ 27
142	57 18.8	3.5	1.020	− 3
143	57 49.1	3.0	1.025	− 4
144	58 13.6	2.5	1.031	− 4
145	58 49.0	3.0	1.028	− 16
146	59 22.2	4.0	1.061	+ 10
147	59 40.0	4.0	1.087	+ 32
148	60 15.6	3.0	1.151	+ 87
149	60 50.2	3.0	1.108	+ 37
150	61 27.3	3.0	1.104	+ 25
151	62 5.3	3.5	1.093	+ 9
152	63 6.7	2.0	1.099	+ 11
153	63 47.2	1.5	1.132	+ 44
154	64 35.0	1.5	1.098	+ 11
155	65 20.9	3.0	1.064	− 24
156	66 2.8	3.5	1.105	+ 18
157	66 20.9	3.5	1.099	+ 12
158	66 36.5	3.5	1.099	+ 13
159	67 11.1	3.0	1.079	− 7
160	68 17.2	3.0	1.076	− 10
161	68 57.8	3.0	1.058	− 28
162	70 59.3	3.5	1.095	+ 12
163	71 28.9	3.5	1.100	+ 18
164	71 43.8	3.5	1.063	− 19
165	71 59.9	3.5	1.075	− 6
166	72 15.5	4.0	1.084	+ 3
167	72 52.9	3.5	1.086	+ 6
168	80 31.8	2.0	1.075	+ 11
169	82 42.0	3.5	1.018	− 41
170	83 3.9	3.5	1.077	+ 19
171	83 18.2	3.5	1.025	− 32
172	83 35.0	3.5	1.048	− 8
173	87 7.4	3.5	1.052	+ 7
174	87 54.3	3.0	1.076	+ 34
175	88 28.6	2.5	1.066	+ 25
176	89 3.0	2.5	1.036	− 3
177	89 48.6	3.0	1.050	+ 14
178	90 9.2	3.5	1.040	+ 5
179	90 25.9	3.5	0.994	− 40
180	90 45.1	3.5	1.030	− 4
181	91 8.7	4.0	1.015	− 18
182	92 17.1	3.0	1.011	− 18
183	92 35.6	3.0	1.004	− 24
184	+92 56.2	3.5	1.018	−0.009

PRINCETON UNIVERSITY OBSERVATORY
May 1923

THE RELATION BETWEEN THE SPECTRA AND THE SIZES OF THE ALKALI METAL ATOMS

By LOUIS A. TURNER

ABSTRACT

Spectra and relative sizes of the positive ions.—A *theory* is developed which gives the relation between the energies and quantum numbers of orbits of valence electrons in atoms differing in size but geometrically similar. It is shown that the *wave numbers* for some of the outer orbits of the four alkali metal atoms Na, K, Rb, and Cs are in fair accord with the assumption that the structure beneath the valence electron, or kernel, is geometrically similar but different in size for the four atoms. The relative sizes of these kernels obtained by this method agree with results obtained by other methods as well as those results agree among themselves. One important consequence of the theory is that for orbits with the same quantum numbers the wave number must be larger for the atom which has the larger kernel. The *quantum numbers* for the *p*, *d*, and *b* terms as given by Bohr in his new theory of atomic structure give the best agreement with this theory.

The radii of the orbits and the field of force around the kernel.—An expression for the maximum radius of a valence electron orbit is derived in terms of the wave number, azimuthal quantum number, strength of the field of the kernel at the outermost point of the orbit, and known constants. For circular orbits, such as the 3*d*- and 4*b*-orbits, both the field-strengths and radii are computed. These results combined with the size ratios of the kernels give curves for the variation of the field-strength as a function of the radius, for the four atoms. Using the equation and these curves, the radii of the atoms in the normal condition are found. They are: Li, 2.38×10^{-8} cm; Na, 2.72; K, 3.45; Rb, 3.61; Cs, 3.94. The application of this method to the 4*b*-terms of the alkaline earth metal spectra shows that the kernel of each must be much smaller than that of the corresponding alkali.

Professor A. Fowler in his *Report on Series in Line Spectra* at the conclusion of the chapter on "Spectra and Atomic Constants" remarks:

> These results are in a sense disappointing. It would seem that the spectra must for the present be regarded as constants of the elements which show no simpler relation to other constants than is shown by some of the constants among themselves. Thus, in the alkali group, the curve connecting atomic weights with melting-points, or that connecting atomic weights and atomic volumes is closely similar to that relating the limits of the subordinate series to atomic weights, and a similar discrepancy is shown by potassium in each case. Again there is no simple relation between the atomic weights and densities in this group of elements, just as there is no simple law connecting the limits of the principal series. It can only be concluded that although the spectra change progressively with atomic weights, atomic volumes, or atomic numbers, the

laws governing the changes are not clearly indicated by any of the foregoing investigations.

Apparently there is need for new work in this field, based on new points of view. In Part I of this paper after assuming atomic models in accord with modern theories of atomic structure, I have shown that the differences between the spectra of the different alkali metals can be related to the differences of the sizes of the structures immediately underlying the valence electron. In Part II I have calculated the radii of some of the electron orbits and the approximate mean field of force in the outer parts of the valence electron's orbits.

I. SPECTRA AND RELATIVE SIZES OF THE POSITIVE IONS

According to Bohr's new theory of atomic structure, and also to Bury's[1] modification of Langmuir's theory, the alkali metal atoms are built of an inner structure of nucleus and electrons surrounded by an outer shell or group of eight, similar in structure for all, and a still more loosely attached single valence electron. It is reasonable to suppose that the great similarity between the spectra of these elements is due to this similarity in structure. Departure from the simple hydrogen spectrum and differences between the spectra of the different elements will arise chiefly because of differences in the outer group of eight. This must be accurately true for lines due to orbits far outside the group of eight, and at least approximately true for all orbits which do not come into the region of strong forces from the second inner group. In what follows I assume, therefore, that for these outer orbits the atom can be considered as a nucleus of charge $+9e$, surrounded by a configuration of eight electrons and a valence electron, and that the configuration of eight has a mean field of force which is geometrically similar but differs in size for the different atoms. This latter assumption can be no more than a good approximation, but it seems reasonable enough and is strongly suggested by the similarity of the spectra of these elements. The change of the outer orbits, and therefore of the spectra, from one element to another of higher atomic number will be due, then, primarily to the change of the size of the struc-

[1] C. R. Bury, *Journal of the American Chemical Society*, **43**, 1602, 1921.

ture of eight, and only indirectly due to the increasing inner complexity. I shall refer to the structure beneath the valence electron as the kernel.

It is necessary to consider the possibility of, and conditions for, geometrically similar atoms of different sizes. Imagine two such atoms geometrically similar but different in size, oriented the same way, and consider the similar orbits of the outer electron. Let atom A' be k times as large as atom A. Since the forces acting upon the outer electron are assumed to be electrical they vary as the inverse square of the distance and since in atom A' all distances are k times as great, all forces and proportional accelerations will be $1/k^2$ times as great. With acceleration $1/k^2$ times as great and distances k times as great, it follows immediately from the theory of dimensions that corresponding velocities in A' will be $1/\sqrt{k}$ times as great as those in A. Further application of the theory of dimensions shows at once that the energies, kinetic and potential, at each point of A' will be $1/k$ of the energies at the corresponding point of A, and the angular momentum will be $\sqrt{k}$ times as great. If orbit A is an actual one with constant energy and angular momentum, orbit A' is a possible one, since it, too, has constant energy and constant angular momentum. The azimuthal quantum numbers for A' will be $\sqrt{k}$ times those for A, since they equal the angular momenta divided by $1/2\pi$ times Planck's constant h. Other quantum numbers will be increased in the same way since they are obtained by dividing quantities of the dimensions of angular momentum by h alone, or by h and a numerical factor. The wave numbers ν of the spectral terms of A' will be $1/k$ times the corresponding wave numbers for A, since these are proportional to the energies.

Consider a sodium atom with the outer electron in some particular orbit and assume for the sake of illustration that a potassium atom has a kernel which is 1.5 times as large. What will be the energy and quantum number of the geometrically similar orbit in the potassium atom? As shown above, the quantum numbers will be $\sqrt{1.5}$ or 1.227 times as great, and the energy, $1/1.5$ or 0.667 times as great. Thus if we take any series of terms, such as the p-terms, of sodium and plot the wave numbers ν as ordinates and

the corresponding total quantum numbers as abscissas we obtain a curve characteristic of sodium which may be transformed into a curve characteristic of potassium by multiplying these ordinates by 0.667 and the abscissas by 1.227. All *p*-orbits of sodium have azimuthal quantum numbers 2. The orbits corresponding to points on the new curve for potassium have azimuthal quantum numbers 1.227 times 2, or 2.454. These new orbits are geometrically similar to the corresponding orbits in sodium not only as regards the parameters of the orbits themselves, but also as regards the relation of the orbits to the kernel. They are dynamically possible orbits, but will not be actual orbits, permitted by the quantum theory, since their azimuthal quantum numbers are not integers. We may designate these "in between," or fractional quantum number orbits as "hypothetical" orbits. The true ratio between the sizes of the sodium and potassium kernels should be that ratio which makes all the actual orbits of sodium transform into such a scheme of hypothetical potassium orbits as will be consistent with its actual orbits.

The difficulty in the immediate application and testing of this idea is that we do not yet know enough about the field of force surrounding the kernel to say just what the curve for orbits with these hypothetical azimuthal quantum numbers should be, i.e., how the hypothetical orbits of fractional azimuthal quantum numbers should be related to the actual orbits, and therefore we have no basis for the acceptance or rejection of various assumed ratios of the sizes.

There is a method of procedure, however, if we restrict our consideration to circular orbits. According to both Bohr and Sommerfeld the $3d$ and $4b$ terms of the four alkali metals Na, K, Rb, and Cs correspond to circular orbits, with azimuthal quantum numbers 3 and 4 respectively, and radial quantum numbers 0.[1] For the Cs atom, for instance, there will be some curve with ν for circular orbits for ordinates and azimuthal quantum numbers for

[1] Different numbers for some of these orbits are given in the table in the appendix to Foote and Mohler's book, *The Origin of Spectra*. Dr. Foote has told me that the book is in error, that the quantum numbers given here are the ones which Bohr believes to be correct.

abscissas. The $3d$- and $4b$-terms give two points on this curve. We have no other orbits of this class actually known for Cs, but these two are but a special pair of the infinite number possible as far as the mechanics of the structure is concerned. These other mechanically possible circular orbits are the "hypothetical" orbits with fractional quantum numbers, some of which can be obtained by transformation of the $3d$- and $4b$-orbits of Na, K, and Rb according to their sizes. If we now imagine the Na, K, and Rb atoms to be expanded so that their kernels will have the size of the Cs kernel, their six circular orbits will become hypothetical circular orbits of the Cs atom. The problem is to find the size factors which will bring all these points on a smooth curve. That is what I have done, essentially, in the following, but by an indirect method which is easier and more certain than it would be to try to do the thing directly.

If we take the equation $\nu = N/n^2$, where N is a constant and plot ν as a function of n, this curve will transform into itself by the foregoing transformation. That is, if we take the values for n and ν for any point on the curve, multiply n by any factor $1/\bar{k}$ and divide ν by k, the new values of ν and n will satisfy the equation and give a new point on the same curve. If we let N be the Rydberg constant, the values of ν corresponding to integral values of n are the wave numbers of the terms of the hydrogen spectrum. I shall refer to values of ν on this curve as ν_H. Take ν for some term of the Na spectrum corresponding to a total quantum number n and find the ratio of this ν to ν_H for the same total quantum number. The geometrically similar orbit in an atom which is k times as large will have a value of ν $1/k$ times as great and a value of n $1/\bar{k}$ times as great. The value of ν_H corresponding to this new n will also be $1/k$ times as great as the old one, so the ratio ν/ν_H will be the same for both of these similar orbits. If we plot these ratios ν/ν_H for Na as a function of n, we get the curve for an atom k times as large, simply by multiplying the values of n by $1/\bar{k}$, leaving the values of ν/ν_H the same. It is more convenient to plot $\log \nu/\nu_H$ because one uses logarithms in the computation of the ratios, and to plot $\log n$ because the multiplication of the values of n by $1/\bar{k}$ is replaced by the addition of $\frac{1}{2} \log k$ to $\log n$, a simple lateral shift in the direction of the abscissas, $\log n$.

The following table gives the available data. The mean between the two d-terms is taken to be the true value for ν, the separation apparently being due to some magnetic effect not considered here.

The range of values of log ν/ν_H is too great for satisfactory plotting on one curve, so I have multiplied them by 10^5 and again taken the log. If there is to be a smooth curve representing log ν/ν_H as a function of log n, there will also be a smooth curve for

TABLE I

	ν	Mean ν	log ν	log ν/ν_H	log($10^5 \cdot$ log ν/ν_H)
			$3d$		
Na	12276	12276	4.08906	0.00317	2.501
K	13467.5 13470.3	13469	4.12934	0.04345	3.638
Rb	14334.3 -missing	14334	4.15637	0.07048	3.848
Cs	16905 16807	16856	4.22676	0.14087	4.148
H	12187	12187	4.08589	.....	.....
			$4b$		
Na	.....	6860.4	3.83635	0.00034	1.531
K	.....	6878.5	3.83749	0.00148	2.170
Rb	.....	6897.6	3.83870	0.00269	2.430
Cs	.....	6932.6	3.84090	0.00489	2.689
H	.....	6855.0	3.83601	.....	.....

log 3=0.477; log 4=0.602

log ($10^5 \cdot$log ν/ν_H) as a function of log n. I have actually plotted the data in the last column of Table I.

By a shift of 0.024 for Rb, 0.044 for K, and 0.142 for Na, seven of the eight points are brought on to a smooth curve, which is almost a straight line, and which gives the relation between wave number and quantum number for the actual and hypothetical circular orbits of Cs. Similar curves, parallel to this, could be drawn for the circular orbits of the other elements. One of the points for Na

alone falls off the smooth curve. No shifts deviating markedly from these would give any reasonable sort of a curve through all of these points, for it is highly improbable that the curve should have many inflections. These shifts give $\frac{1}{2} \log k$ where k is the ratio of the size of the Cs kernel to that for the other atom concerned.

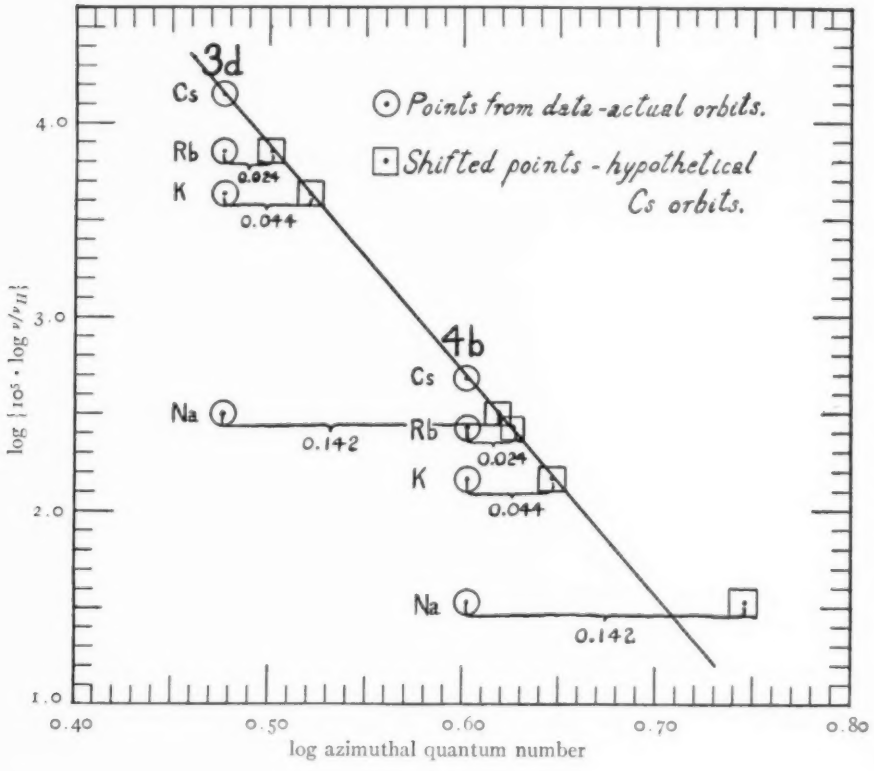

FIG. 1

The values of k are as in Table II. I shall reserve discussion of them until I have found another set of values in a similar manner.

One important result of this theory, even if it should be only a rough approximation, is that the values of ν must be *larger* for the same kind of an orbit in an atom of larger kernel. Otherwise, a shift of this sort would be in the opposite direction from that above and would indicate that the size of the atom increased

with decreasing atomic number. Thus, contrary to Sommerfeld's assignment of quantum numbers, the $2p$-terms of the alkalis cannot all have the same total quantum numbers, for they *decrease* in value as one goes to atoms of higher atomic number. The same can be said for the $3d$-terms in the enhanced doublet spectra of the alkaline-earth elements. The total quantum number for the first p-term will have to be at least one greater for each atom as one goes up the scale of atomic numbers. That is precisely the way that Bohr has assigned the numbers in the new theory, according to which the azimuthal quantum number for all the p-terms is 2, and the total quantum numbers for the first terms for each atom are as follows: Na. 3; K, 4; Rb, 5; Cs, 6. These are indicated in his notation, which I shall use, as 3_2, 4_2, 5_2, and 6_2 orbits.

TABLE II

	$\frac{1}{4}\log k$	$k=R_{C_s}/R_x$
Rb.....	0.024	1.12
K.....	0.044	1.23
Na.....	0.142	1.92

It is not possible to test this size theory in just the same way with the p-orbits as with the $3d$- and $4b$-orbits, because the orbits, not being circular, lie in regions of variable force, and we do not know enough about the nature of this variation to properly interpret the relations between the "hypothetical" orbits. Failing this, we can still consider some particular cases if we make the plausible assumption that there should be a relation between ν/ν_H and total quantum number for such orbits as have the *same ratio of azimuthal to total quantum number*. In a hydrogen-like atom, this would mean geometrical similarity of orbits, and there would be no new assumptions involved in treating the case as we treated the case of circular orbits. In the cases of the alkali metals, the additional assumption involved is that for orbits with the same ratio of azimuthal to total quantum number the variations of wave number depend in a continuous manner on the azimuthal quantum number for each atom. The curves representing this relation must shift into one another by the size transformation used above, since the

change of size does not alter the *ratio* between the two quantum numbers.

As an example, we can take the 6_2 and the 9_3 terms for all the elements. This will be the first p-term for Cs, the second for Rb, the third for K, and the fourth for Na, and the seventh d-term for all. For all of these, the ratio of the total to the azimuthal quantum number is 3. The same argument applies to these orbits as to the

TABLE III

	ν	Mean ν	log ν	log ν/ν_H	log(10³·log ν/ν_H)
	6_2				
Na	4151.3 4152.8	4152.0	3.618	0.134	2.127
K	6001.2 6009.3	6005.2	3.778	0.294	2.468
Rb	9896.6 9974.1	9935.3	3.996	0.512	2.709
Cs	19672.3 20226.3	19949.3	4.300	0.816	2.912
H	.....	3047	3.484	.....	.....
	9_3				
Na	.....	1357.2	3.132	0.001	0
K	.....	1433.6	3.156	0.025	1.397
Rb	.....	1464.1	3.165	0.034	1.544
Cs	.....	1506.1	3.177	0.046	1.662
H	.....	1354	3.131	.....	.....

log 2=0.301; log 3=0.477

circular ones before, and the same process of finding the shifts to bring all the points on one curve can be applied. It should be emphasized that this does not necessarily imply that the 6_2 and 9_3 and other such orbits in any one atom are necessarily geometrically similar, but simply that, in the nature of the case, we would expect to find some curve giving the value of ν as a function of azimuthal quantum number for all such orbits.

The data are given in Table III. Figure 2 shows the curve obtained.

The shift for Rb is 0.023, and that for K is 0.052. The upper Na point comes on to the curve by a shift of 0.098, but the lower could not be made to lie on the curve. As in Figure 1, these shifts bring all but one point on to a smooth curve, but the agreement

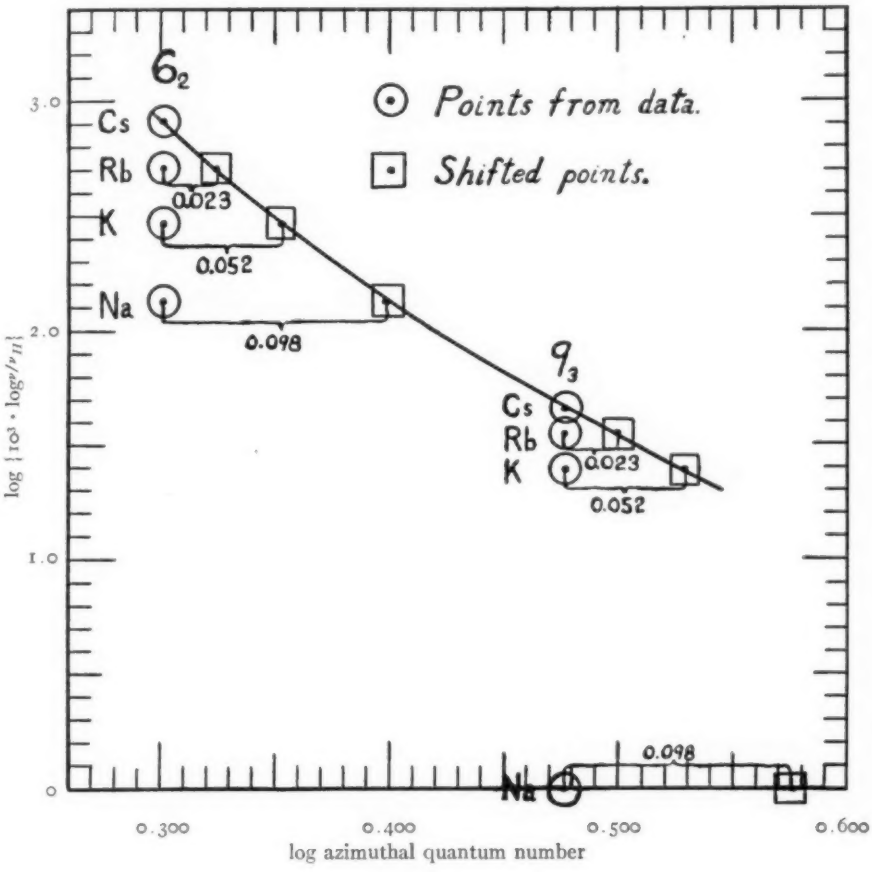

FIG. 2

of these shifts with those calculated from Figure 1 is not very impressive, unless one considers that it is rather remarkable that there should be any agreement at all for data picked in this special way unless there was some truth in the theory governing the choice. There are two good reasons for expecting failure of perfect

agreement. One is the probable deviation of the fields of the kernels from exact geometrical similarity even up to considerable distances from the center. The other is the fact that in the p-orbits the electrons are thought to penetrate much nearer the central structure than in the d-terms, probably near enough so that the assumption of a nucleus of point charge $+9e$ is no longer strictly valid. In this connection it is noted that the best agreement between the shifts in Figures 1 and 2 is found for those elements whose inner structure is, relatively, least different.

The same kind of a curve can be made for the 8_2- and 12_3-orbits. The available data are not complete, however. Using the shifts

TABLE IV

	Mean ν	log ν	log ν/ν_H	log ($10^3 \cdot \log \nu/\nu_H$)
	8_2			
Na	2150.2	3.332	0.098	1.991
K	2779.4	3.444	0.210	2.322
Rb	3845.1	3.585	0.351	2.545
Cs	5655.0	3.743	0.509	2.706
H	1714	3.234	.....	.....
	12_3			
Na	761.7	.....	.....	.....
K	.....	.....	.....	.....
Rb	.....	.....	.....	.....
Cs	825.8	2.917	0.035	1.544
H	761.0	2.882	.....	.....

from the last curve, these points are brought into line. The 12_3 for Na (not plotted) would be badly off again. Figure 3 shows this curve.

The results of the two methods are as in Table V. The average value of $\frac{1}{2} \log k$ is used to get the ratios of the sizes. The corresponding ratios of Bragg's radii of the ions in crystals and also of the radii of the nearest rare gas atoms as determined from (1) the constant b in Van der Waal's equation and (2) the mean free paths are given for comparison. Very close agreement would not be expected here because there is no theory adequate to relate the region belonging to an ion in a crystal to its actual size or to give an idea of its probable deformation in the structure of a crystal, neither do we

know just what relation the size of the sphere of influence of a rare gas atom bears to the actual size of the atom, or to that of the kernel of the following alkali metal.

We see, therefore, that not only does this theory lead to a fairly good co-ordination of the spectra of the alkali metals in terms of the assumed sizes of their atomic kernels, but also that these assumed

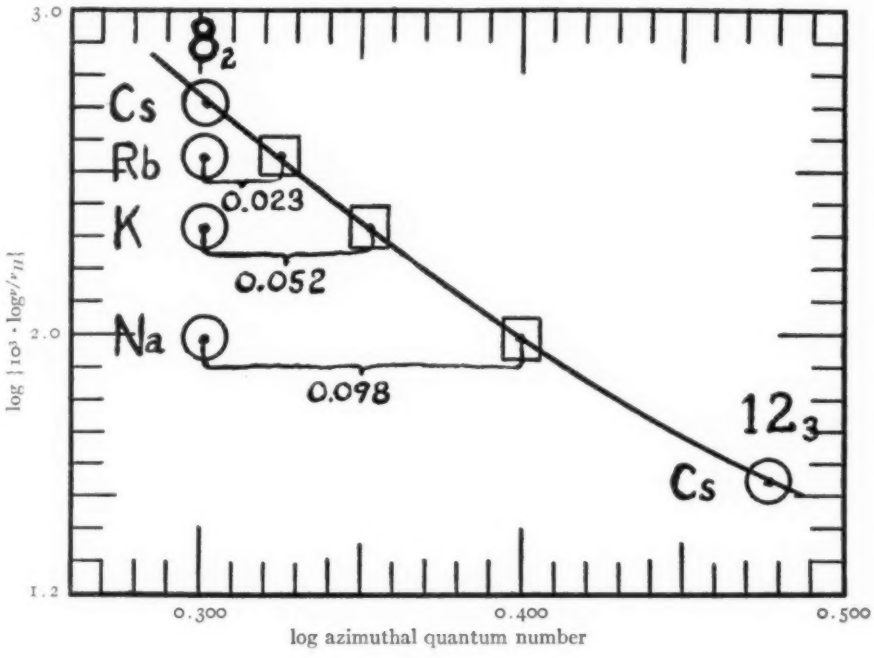

FIG. 3

sizes are reasonable ones in view of the meager data available for comparison.

This theory may be used to test the correctness of the quantum numbers assigned to the various spectral terms—a matter of interest because Bohr's and Sommerfeld's numbers differ. Such a test cannot be made from the spectra themselves, since a difference in the assignment of quantum numbers can be compensated by an equal opposite change in the value of the other term constants, whose values have not hitherto been known by any independent method. I have tried various other choices of quantum numbers

and have found no case in which the predictions of the present theory are satisfied with as good a degree of consistency as is shown in the instances cited above. These results are, therefore, a confirmation of the numbers assigned by Bohr.

It can be concluded from this work that the size of the kernel is very important in determining the difference between the homolo-

TABLE V

	$\frac{1}{2}\log k$		R_{C_3}/R	R_{C_3}/R	$R_{xe}/R(Kr, A, Ne)$	$R_{xe}/R(Kr, A, Ne)$
Rb.....	0.024	0.023	1.12	1.06	1.09	1.30
K.....	0.044	0.052	1.25	1.14	1.20	1.325
Na.....	0.142	0.098	1.74	1.34	.....	1.85
	$3d$, $4b$	6_2, 9_3 8_2, 12_3		Bragg	Van der Waal	m.f.p.

gous terms of the spectra of the alkali metal atoms and that Bohr's assignment of the quantum numbers for these terms is in agreement with this theory.

II. THE RADII OF THE ORBITS AND THE FIELD OF FORCE AROUND THE KERNEL

According to Bohr's new theory the valence electron in an alkali metal atom describes an orbit which is much like a rotating ellipse. At its outermost position it is thought to be at quite a distance from the center of the atom as compared with the distances between the other parts of the atom. At this outermost position its velocity vector must lie in a position perpendicular to the radius vector. Call this radial distance r, the energy E, the velocity v, the mass m, the electronic charge e, Planck's constant h, and the azimuthal quantum number k. Denote the potential energy of the electron by $-pe^2/r$. e^2/r is the potential energy of an electron at a distance r from a charge of $+e$, so p is the potential energy factor or apparent charge on the equivalent point nucleus. In general p is a function of r in virtue of the second and higher order terms in the expression for the potential due to any assemblage of charges, and it approaches the value unity to the extent to which the potential can be consid-

ered as due to a single point charge. From the conditions that the energy and the angular momentum are constant one gets the two equations

$$mvr=\frac{kh}{2\pi} \qquad E=\tfrac{1}{2}mv^2-p\frac{e^2}{r}$$

which combine to give

$$E=\tfrac{1}{2}m\frac{k^2h^2}{4\pi^2m^2r^2}-\frac{pe^2}{r} \qquad \text{or} \qquad r^2+p\frac{e^2}{E}r-\frac{k^2h^2}{8\pi^2mE}=0.$$

Solving for r gives

$$r=\frac{-p\dfrac{e^2}{E}\pm\sqrt{\dfrac{p^2e^4}{E^2}+\dfrac{k^2h^2}{2\pi^2mE}}}{2}=\frac{-pe^2}{2E}\left(1\pm\sqrt{1+\frac{k^2h^2E}{2\pi^2me^4p^2}}\right).$$

This equation can be put in a form more convenient for application by putting in different constants as given by Bohr's theory of the hydrogen atom. The radius of a normal hydrogen atom is $a_0=\dfrac{h^2}{4\pi^2me^2}$. The Rydberg constant, $N=\dfrac{2\pi^2me^4}{h^3c}$. Their product $a_0N=\dfrac{e^2}{2hc}$. If ν is the wave number for a spectral term or stationary state of the atom, $E=-hc\nu$. Making these substitutions we get

$$r=\frac{pe^2}{2hc\nu}\left(1\pm\sqrt{1+\frac{-k^2h^3c\nu}{2\pi^2me^4p^2}}\right)=p\frac{N}{\nu}a_0\left(1\pm\sqrt{1-\frac{k^2\nu}{p^2N}}\right).$$

It is apparent from the derivation that these two values of r are the same as the aphelion and perihelion radii for an elliptical orbit in a hydrogen-like atom with a charge of $+pe$ on the nucleus. These two radii, in terms of the semimajor axis and eccentricity of the ellipse are $(1+e)a$ and $(1-e)a$. Hence the expression under the radical sign gives the eccentricity. It is possible to find the values of p and r for a circular orbit, since the eccentricity is equal to 0. The radical can be set equal to 0 and the value of p thus obtained used in finding r. By this method the following values for p and r have been calculated.

It is seen that these orbits have approximately the radii $9a_0$ and $16a_0$ of the corresponding hydrogen orbits, the agreement

being more exact for the outer orbit and for the atom with the smallest nucleus. Under the same conditions the potential energy factor p is nearly unity, but it increases rapidly at distances nearer the nucleus, especially for the larger atoms.

Consider one of the atoms where the value of p is a certain quantity at a distance r from the center. What will be the value of p at the corresponding distance from the center in an atom k times as large? The corresponding distances will be kr and the

TABLE VI

$a_0 = 0.532 \cdot 10^{-8}$ cm

	Mean ν	p	r	r (cm)	Mean ν	p	r	r (cm)
	$3d$-Terms				$4b$-Terms			
Na....	12276	1.0037	$8.97\,a_0$	$4.77 \cdot 10^{-8}$	6860.4	1.0004	$15.99\,a_0$	$8.50 \cdot 10^{-8}$
K.....	13469	1.0513	$8.57\,a_0$	4.61	6878.5	1.0017	$15.98\,a_0$	8.49
Rb....	14334	1.0845	$8.29\,a_0$	4.40	6897.6	1.0031	$15.95\,a_0$	8.48
Cs....	16856	1.1761	$7.64\,a_0$	4.06	6932.6	1.0057	$15.90\,a_0$	8.46

new potential energy $1/k$ times the former. Let primed symbols refer to the larger atom. Then

$$E = -p\frac{e^2}{r} \qquad E' = -p'\frac{e^2}{r'}$$

$$E' = \frac{E}{K} \qquad r' = rK \qquad \therefore \quad p' = p$$

Hence p is the same for both, as would be expected from the principle of dimensions. If then we plot values of p as ordinates and values of r as abscissas there will be a curve for each atom and these curves will be transformable one into the other by multiplying the radii by the proper size ratio factors as found before. For reasons as before, it is better to plot log $\{(p-1)\cdot 10^4\}$ and log r. The curves will then all be the same but shifted along the log r axis. The data plotted are as in Table VII.

Figure 4 shows the curves obtained using the size factors found from the d and b terms in Part I. These curves give, within limits, the way in which the potential energy factor varies with distance from the center for the various atoms. They permit a calculation

of the aphelion distances of any orbits which lie within these limits of distance.

Fues[1] has shown that the spectrum terms of Na can be accounted for by assuming that the electron orbits are in a central field of force which varies with distance from the center in approximately the same way as the field about a central nucleus of charge $+9e$ surrounded by eight electrons at the corners of a cube varies along a radius perpendicular to a cube face. It is of interest to see whether the field of force about the kernel which these curves give is of that sort. The equation for the potential energy at a distance

TABLE VII

	$\log\{(p-1)\cdot 10^4\}$	$\log r$	$\log\{(p-1)\cdot 10^4\}$	$\log r$
	d		*b*	
Na.....	1.568	0.952	0.602	1.203
K.....	2.710	0.933	1.230	1.203
Rb.....	2.927	0.918	1.491	1.202
Cs.....	3.246	0.883	1.756	1.201

r from the center along a line perpendicular to the face of the cube of side $2a$ is

$$V=-\frac{9e^2}{r}+\frac{4e^2}{\sqrt{(r-a)^2+2a^2}}+\frac{4e^2}{\sqrt{(r+a)^2+2a^2}}.$$

Expanding in powers of a/r by the binomial theorem gives

$$V=-\frac{e^2}{r}\left\{1+\frac{84}{3}\frac{a^4}{r^4}\right\} \text{ plus higher power terms negligible for small } a/r.$$

To a first approximation then the expression for p is

$$p=1+\frac{84}{3}\frac{a^4}{r^4}.$$

It follows that

$$\log\{10^4(p-1)\}=\log\left\{\frac{84}{3}\cdot 10^4\cdot a^4\right\}-4\log r$$

so the $\log\{10^4(p-1)\}-\log r$ curves should be straight lines of

[1] E. Fues, *Zeit. für Phys.*, **11**, 6, p. 364, 1922.

slope -4. The curves are a fair approximation to straight lines. The slope of the line drawn with the Cs curve on Figure 4 is -5.9. The value of p falls off more rapidly with increasing ν than for the cube. The slope of the straight line through the two Na points, however, is -3.85, in fair agreement with Fues's assumption. Apparently the assumption of geometrical similarity of the field

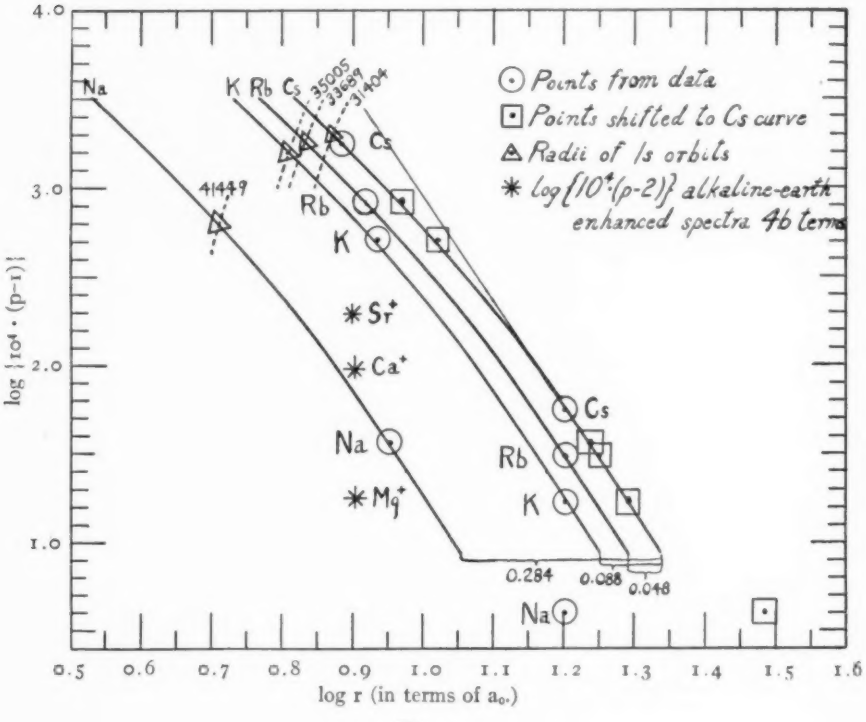

FIG. 4

about the kernels is a fair approximation for K, Rb, and Cs, but is not very accurate when Na is compared with one of the others.

With these curves it is now possible to find the radii of the electron orbits for the normal state of the atom, corresponding to the 1*s* spectroscopic level, since it is found that the electron comes into the region of the field represented on these curves. The quickest way to do this is to take a value of ν for a 1*s*-term, such as 44149 for Na and by use of the equation $r = p\frac{N}{\nu}\left(1+\sqrt{1-\frac{k^2}{p^2}\frac{\nu}{N}}\right)a_0$

find corresponding pairs of p and r for this ν. That is, taking a fixed value for ν, assign various values to p and calculate the corresponding values of r. Plot these points on the same sheet with the $p-r$ curve for the Na atom and the intersection of the two curves will give the desired radius, since the values of p and r at that point fit both the field of the atom and the particular value of ν. I have plotted these curves as dotted lines on Figure 4, using the logarithms of $\{10^4 \cdot (p-1)\}$ and r. The radii obtained for the atoms are as follows. The one for Li is obtained on the assumption that $p=1$. The curve for $\nu=41449$ is steep so that the value of $\log r$ for the intersection would not be changed much even though the curve for the field around the Na kernel may have a smaller slope.

TABLE VIII

Metal	$\log r$	r	r(cm)
Li	.....	$4.48a_0$	$2.381 \cdot 10^{-8}$
Na	0.709	5.12	2.721
K	0.809	6.45	3.43
Rb	0.831	6.78	3.61
Cs	0.870	7.41	3.94

These values should be closely proportional to the radii of the atoms in the gaseous state. Unfortunately there are no available data on these radii.

Crystal analysis experiments indicate that metallic Na and K crystallize with the atoms arranged in a body-centered cubic structure with cube edges of 4.30 and 5.20 A respectively. The distance between nearest atoms in such structures are 3.72 and 4.50 A. These orbital radii are of the same order of magnitude as the internuclear distances in the crystal structures, making it seem highly improbable that in the crystal the atoms are in the neutral state, each with its own valence electron. It seems to indicate that the atoms must be present as positive ions, the electrons fitting into the structure between the ions rather than following their usual orbits.

Making the assumption that the $4b$-terms of the enhanced spectra of the alkaline-earth metals correspond to circular orbits of quantum number 4, it is possible to calculate r and p for these

orbits as above. This term for Ba shows an anomalous behavior in that it is less than the one for Sr, so I have not used it. It may correspond to an elliptical orbit of quantum numbers 5_4. In this case, the value of $(p-2)$ represents the change of the field because of the extended structure of the nucleus. I have plotted corresponding values of $\log \{10^4(p-2)\}$ and $\log r$ for these terms, on Figure 4. They show clearly that the kernel is much smaller for

TABLE IX

Element	$\nu-4b$	p	r	$\log \{10^4(p-2)\}$	$\log r$
Mg+.....	27467.4	2.0018	7.99 a_0	1.255	0.902
Ca+.....	27686	2.0097	7.96 a_0	1.986	0.901
Sr+.....	27960.4	2.0198	7.91 a_0	2.297	0.898
Ba+.....	25288.8	.....	.....	.....	.....

any alkaline-earth metal atom than for the corresponding alkali metal atom. The curve for any alkali metal atom lies to the left of the curve for any one which has a larger kernel. Each of these alkaline-earth metal points lies well to the left of the curve for the corresponding alkali metal. Such shrinkage would be expected with the increase of nuclear charge. The data are as in Table IX.

In conclusion, I wish to express my indebtedness to Professor K. T. Compton for valuable advice and help in this work.

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WAVE-LENGTHS OF LINES IN THE SPECTRA OF STARS OF CLASS Me[1]

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ABSTRACT

Wave-lengths of emission and absorption lines in the spectra of numerous long-period variables.—An intercomparison of the measurements of the *bright lines* Hβ, Hγ, Hδ, Hζ, λλ 4571 Mg, 4308 Fe, 4202 Fe, and 3905 Si, shows that the relative wave-lengths of these lines are nearly the same in the variables as in the laboratory. An additional list of 14 bright lines is given, each of which has been measured on 5 or more spectrograms. Most of these lines have not been identified. Ninety-five *absorption lines* are listed, over half of which are identified with low temperature lines of metals. The temperature of the reversing layer near the time of maximum light appears to be about 2200°.

As by-products of an investigation of the radial velocities of long-period variables,[2] numerous wave-length determinations of the lines used for velocity and of other lines measured incidentally have been accumulating. Since a part of the data for class S stars has been published,[3] the present paper will deal principally with lines in spectra of class Me.

The measurements reported here were all made from spectrograms taken with one-prism dispersion. The Mount Wilson spectrograms upon which are based all of the results for the absorption lines, and most of those for the emission lines, have a dispersion as follows: at Hβ, 56.6; at Hγ, 36.1; at Hδ, 28.2 A per mm. Measurements with higher dispersion are of course desirable, but the observational work involved would be heavy, because of the many long exposures required to go over the ground already covered by one-prism observations. The one-prism results are therefore published now, as it will probably be several years before they are supplanted by measurements with higher dispersion. A list of the stars observed, with the dates of observation and various details, will be found in *Mount Wilson Contributions* No. 264.

[1] *Contributions from the Mount Wilson Observatory*, No. 265.

[2] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 215, 1923.

[3] *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.

EMISSION LINES

Table I contains a list of the emission lines used for radial velocity, the wave-lengths being recent laboratory determinations.

In order to ascertain whether the wave-lengths of these lines have the same relative values in the variable stars as in the labora-

TABLE I

LABORATORY WAVE-LENGTHS OF EMISSION LINES

I. A.	Identification
3835.36	Hη
3889.05	Hζ
3905.51	Si
3970.08	Hε
4101.74	Hδ
4202.03	Fe
4307.91	Fe
4340.47	Hγ
4571.11	Mg
4861.33	Hβ

TABLE II

AGREEMENT OF EMISSION LINES

	Class Me						Class Se					
	100-inch		60-inch		Ann Arbor		100-inch		60-inch		Ann Arbor	
	km	No.	km	No.	km	No.	km	No.	km	No.	km	No.
Hβ−Hγ	−0.5	40	−1.2	18	−0.6	28	−0.3	43	−5.3	19	+2.	7
Hγ−Hδ	+0.4	180	−0.6	75	+2.8	107	+4.8	31	+0.4	11	.....	.....
Hδ−Hζ	−5.0	14	−5.	2	−10.0	23	.....	.....	.....	.....	.....	.....
Hγ−4308	−3.9	21	−3.4	5	.....	.....	−1.8	5	.....	.....	.....	.....
$\frac{Hγ+Hδ}{2}$−4202	−0.6	33	+1.0	10	− 5.8	5	−2.	2	.....	.....	.....	.....
Hγ−4571	−0.6	20	−5.2	6	.....	.....	−1.	2	.....	.....	.....	.....
Hζ−3905	−2.4	7	−5.5	2	+ 6.4	5	.....	.....	.....	.....	.....	.....

tory, data showing the mean differences in kilometers between velocities from individual lines have been collected in Table II. The results from the Ann Arbor and Mount Wilson observations (two spectrographs for the latter) are given separately. The figures following a difference indicate the number of plates upon which the difference depends.

Slight systematic differences between different lines may arise from various observational causes such as atmospheric dispersion, photographic or bisection errors depending on line intensities,

TABLE III

WAVE-LENGTHS OF EMISSION LINES IN SPECTRA OF CLASS Me

I.A.	P.E.	No. Spectrograms	No. Stars	Probable Identification
3852.63	±0.03	5	1	
3977.78	.03	6	1	
4030.51	.03	8	3	Mn 4030.76
4103.03	.01	37	15	Si 4102.95
4138.65	.01[5]	15	9	
4178.82	.01[5]	27	12	
4215.74	.04	6	4	
4233.35	.01	38	16	
4372.58	.01	19	8	
4375.84	.02	16	5	Fe 4375.93
4458.83	.03	10	5	
4511.46	.02	18	7	
4521.54	.04[5]	5	3	
4578.84	±0.03	6	4	

NOTES TO TABLE III

4030.51 This line first appears as a bright edge on the violet side of the low temperature manganese line 4030.76. The identification is probably correct in spite of the considerable discrepancy between the stellar and the laboratory wave-length.

4103.03 A close companion to Hδ. Its distance from Hδ as measured directly is, in the mean, 1.28 A. When added to the wave-length of Hδ, 4101.74, this gives 4103.02, in very good agreement with the wave-length depending on the mean velocity displacements. The measured wave-length may be too great on account of the close proximity of the very strong Hδ line. King's[1] wave-length of the silicon line to which the stellar line may correspond is 4102.95.

4233.35 This can scarcely be the enhanced iron line 4233.14.

4375.84 R Leonis gives a consistently low value for this line. Eight spectrograms of four other stars give a mean value of 4375.94.

4458.83 Marked doubtful on several measures. It is near the head of a titanium oxide band.

4511.46 Occurs also in class Se spectra.

4521.54 Occurs also in class Se spectra.

deviation of certain comparison lines from their adopted values, etc. As errors from several of these causes are necessarily small when the lines are close together, the hydrogen lines nearest to the metallic lines have been chosen for comparison.

[1] *Publications Astronomical Society of the Pacific*, **35**, 171, 1923

The hydrogen lines Hβ, Hγ, and Hδ are in substantial agreement in the M-type stars. The effective wave-length of Hζ appears to be about 0.1 A greater than the standard laboratory value. This divergence may or may not be inherent in the stellar rays. In the S-type stars Hδ shows a small displacement toward the violet with respect to Hγ.

The line λ 4308, if correctly identified as the iron line λ 4307.91, is shifted toward longer wave-lengths by a few hundredths of an angstrom; λ 4202 agrees very closely with the assumed laboratory value Fe 4202.03; and λ 4571 Mg and λ 3905 Si agree, within reasonable limits, with their laboratory values. These identifications have recently been called in question by Baxandall.[1] The results in Table II are probably not to be considered as furnishing a conclusive answer to this question, but in so far as they have a bearing on the matter I think they tend to support the assumed identifications.[2]

A number of bright lines have been measured in addition to those in Table I. They have not been used for finding the radial velocity, but their wave-lengths have been deduced on the assumption that they have velocity displacements corresponding to the mean adopted bright-line velocity for each plate. Those lines which have been measured on five or more spectrograms have been collected in Table III.

ABSORPTION LINES

The wave-lengths of the absorption lines which were used for velocity determinations will be found in Table II of *Contribution* No. 264. These wave-lengths have recently been corrected by applying the mean systematic residuals from all the measures, and thus modified appear in Table IV, together with other lines which were not used at all for determinations of velocity but whose wave-lengths were deduced from the measurements. These additional lines were included when five or more determinations were available, provided the probable error from a single spectrogram did not

[1] *Observatory*, **46**, 82, 1923.

[2] The measurements by Adams and Joy of the bright lines of o Ceti (*Publications Astronomical Society of the Pacific*, **35**, 168, 1923) are in good general agreement with the results in Table II.

exceed 0.08 A. For those cases in which the adopted value differs from the laboratory wave-length of the line to which it is believed to correspond, the chemical symbol has been placed in parentheses.

TABLE IV

CORRECTED WAVE-LENGTHS OF ABSORPTION LINES IN SPECTRA OF CLASS Me

3998.61 (V)	4147.68 Fe	4306.05
4020.56	49.76	07.75 (Fe)
26.95	52.32	14.66
30.83 (Mn)	73.99	25.89 (Fe)
33.07 Mn	74.37	30.02 V
34.59 (Mn)	79.56 (V)	32.74 (V)
35.87 (Mn)	90.73	37.50 (Cr)
41.29 (Mn)	94.90	44.42 (Cr)
44.21 (K)	97.00	47.21
45.87 (Fe)	4198.53	68.05
54.87	4200.08	75.93 Fe
57.10	02.10 (Fe)	79.38 (V)
63.76 (Fe)	03.63	83.70 (Fe)
76.36	04.94	84.24 (Fe, V)
77.81 (Sr)	06.74 (Fe)	84.71 V
88.16	15.82 (Sr, Fe)	89.5 (Fe, V)
90.50 V	26.73 Ca	91.67
4092.47 (V)	32.59 (V, Fe)	4395.22 V
4105.05 (V) ?	34.10	4404.85 (Fe)
09.65 (V)	54.30 (Cr)	08.3 (V, Fe)
11.79 (V)	58.34 (Fe)	12.24
13.69	60.48 Fe	21.95
15.19 (V)	74.80 Cr	27.30 Fe
16.58 (V)	82.73 (Fe)	37.85 V
18.60	85.84 (Ti)	55.25
21.62	87.48 (Ti)	62.03
23.60 (V)	89.59 (Cr)	82.14
28.00 V	91.44 (Fe)	94.37
29.78	94.26 (Fe)	4496.57
32.07 (V)	4299.07	4548.61
34.39 (V)	4300.79	80.23
39.93 Fe	02.67	

The number of discrepancies amounting to three or more times the probable error is surprising. These large differences do not necessarily mean that the identification is wrong, as they may be due to systematic displacements of the lines, caused, for example, by

a partially bright edge or a close invisible companion line. It is possible, of course, that some lines have been erroneously identified. These may be detected later by means of more data, especially by measurements with higher dispersion. In general support of the identifications in Table IV, it may be said that groups of lines which are shown by laboratory work to belong together are fairly consistently represented in the table.

Among the most prominent absorption lines in class Me spectra are the low temperature lines of iron, vanadium, manganese, chromium, calcium, and strontium. The temperature of the reversing layer near the time of maximum light, as indicated by a comparison with King's studies of the behavior of lines in the electric furnace, is about 2200°.

It is hoped at some future time to make a detailed study of the lines present in the spectra of long-period variables, especially of their relationship to temperature, ionization, and spectral series.

MOUNT WILSON OBSERVATORY
May 1923